INFINITE SETS

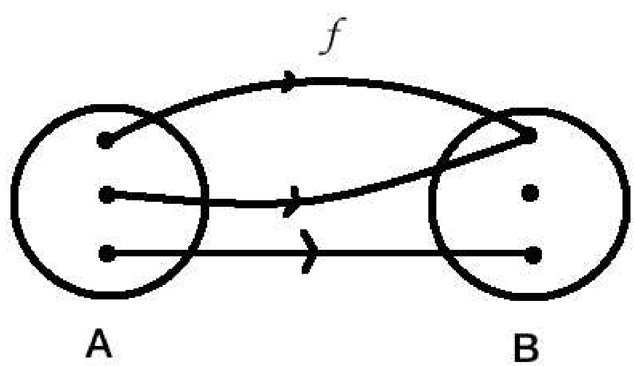
BEGINNERS 10/26/2014

We have seen that

- If two finite sets A and B have the same number of elements, there is a function $f : A \to B$ that is both one-to-one and onto.
- If there is a function $f : A \to B$ that is both one-to-one and onto, the number of elements in A and B is the same.

We will call two such finite sets *equivalent*.

(1) Consider a function $f : A \to B$ defined by the diagram below



Is this function defined?

Is it onto?

Is it one-to-one?

Can you conclude that sets A and B have different numbers of elements?

(2) Construct a function from the set $A = \{1, 2, 3, 4, 5\}$ to the set $B = \{a, b, c, d, e\}$ which is not onto and/or not one-to-one.

Can you conclude that A and B are not equivalent?

(3) How can we compare the size of two *infinite* sets using functions?

Definition 1. Two sets A and B (finite or infinite) are *equivalent* if there is a function $f : A \to B$ such that

- f is one-to-one;
- f is onto;

Note that if we want to show that A and B are not equivalent, we have to prove that no such function exists.

Definition 2. A set A is called *countable* if it is equivalent to the set $\mathbb{N} = \{1, 2, 3, 4, 5...\}$.

(1) Explain the name *countable*.

(2) Enumerate elements of $\mathbb{Z} = \{... - 3, -2, -1, 0, 1, 2, 3...\}$ starting in the following way $0 \mapsto 1 \mapsto -1 \ldots$.

Are the sets $\mathbb{Z}$ and $\mathbb{N}$ equivalent? Does a function $f : \mathbb{Z} \to \mathbb{N}$ which is both onto and one-to-one exist? Can you define it?