LAMC Beginners' Circle **October 13, 2013**

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Warm-up

The following problem was communicated to me by one of our students, Arul Kolla.

Problem 1 *Use four fours to make fifty.*

Note 1 *Using four fours, you can make any number from zero to 50. Try it at home!*

Problem 2 *Ten numbers are written in a line. The first number equals seven. The sum of any three consecutive numbers equals fifteen. What is the last number?*

Back to place-value numerals

Example 1 *Represent* 46_{10} *as a hexadecimal.*

In the decimal system, $46 \div 16 = 2$ *rem* 14*. Hence, the following is true.*

$$46_{10} = 2_{10} \times 16_{10} + 14_{10} = 2e_{16}$$

Problem 3 *Represent* 31_{10} *as a hexadecimal.*

Example 2 *Represent* 571_{10} *as a hexadecimal.*

In the decimal system, $571 \div 16 = 35$ *rem* 11*. In other words,* $571 = 16 \times 35 + 11$*. Now,* $35 > 16$*, so we need to continue. The formula* $35 = 2 \times 16 + 3$ *leads to the following.*

$$571 = 16 \times (2 \times 16 + 3) + 11 = 2 \times 16^2 + 3 \times 16 + 11$$

So, here is the answer.

$$571_{10} = 23b_{16}$$

Problem 4 *Represent* 967_{10} *as a hexadecimal.*

Problem 5 *Represent* $3,258_{10}$ *as a hexadecimal.*

Problem 6 *Represent the trinary number* 12121_3 *as a hexadecimal.*

Problem 7 *Represent* 176_{10} *as an octal number.*

Problem 8 *What is the number* 10_b *in any base* b*?*

Problem 9 *What is the number* 101_b *in any base* b*?*

Problem 10 *Oleg looked at the sheet of paper Anton was working with and noticed the following computation.*

$$13^2 = 171$$

"This cannot be true!" said Oleg. "The decimal system is boring," responded Anton, "I computed this square in a base different from ten." What was the base of the place-value system Anton used for the above computation?

Problem 11 *Use Egyptian multiplication (and, if needed, the table on page 9 of the first handout) to compute the following product.*

𓍢		𓍢 𓍢
∩ ∩ ∩	*times*	∩ ∩ ∩ ∩ ∩

Problem 12 *Perform the following long subtraction of the trinary numbers without switching to the decimals.*

$$\begin{array}{rrrrrrr} & 1 & 2 & 0 & 1 & 2 & 1 \\ - & & 1 & 2 & 2 & 1 & 2 \\ \hline \end{array}$$

Then convert all the three numbers to the decimal form and check your answer.

Problem 13 *Perform the following long addition of the hexadecimal numbers without switching to the decimals.*

$$\begin{array}{r} a\ b\ c \\ +\ d\ e\ f \\ \hline \end{array}$$

Then convert all the three numbers to the decimal form and check your answer.

Problem 14 *Complete the binary multiplication table below. Use it to perform the following long multiplication without switching to decimals.*

×	0	1
0		
1		

$$\begin{array}{r} 1\ 0\ 1\ 1 \\ \times \quad 1\ 1\ 0 \\ \hline \end{array}$$

Then convert both factors and the product to the decimal form and check your answer.

Problem 15 *Complete the trinary multiplication table below. Use it to perform the following long multiplication without switching to decimals.*

$\times$	0	1	2
0			
1			
2			

$$\begin{array}{r} 1\ 2\ 0\ 2 \\ \times \quad 1\ 1\ 2 \\ \hline \end{array}$$

Then convert both factors and the product to the decimal form and check your answer.

Problem 16 *Complete the octal multiplication table below.*

$\times$	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

Use the table to perform the following long multiplication of octal numbers without switching to the decimals.

$$\begin{array}{r} 7\ 6 \\ \times\ \ 4\ 5 \\ \hline \end{array}$$

Find the decimal representations of the factors and of the product and check the answer.

Problem 17 *Use the hexadecimal multiplication table at the end of the second handout to perform the following long multiplication without switching to the decimals.*

$$\begin{array}{r} d\ 3 \\ \times\ 7\ c \\ \hline \end{array}$$

Then convert the factors and the product to the decimal form and check your answer.

Problem 18 *Use long division to solve the following problem (with the remainder) without switching to decimals.*

$$1101101_2 \div 11_2$$

There are two ways to check whether your solution of Problem 18 is correct.

Problem 19 *Multiply the quotient from Problem 18 by the denominator and add the remainder. See if you get back the numerator this way. Do not switch to decimals.*

Problem 20 *Find the decimal representations of all the relevant numbers from Problem 18 and check the correctness of its solution by performing long division in the decimal system.*

From hexadecimals to binaries and back again

Sixteen is a power of two. That is why there exists a very fast and efficient way of switching back and forth between the binaries and hexadecimals that does not involve the decimals.

Example 3 *Represent the number $f3$ in the binary form. (Note that we do not need the base subscript for this number – the digit f clearly indicates hexadecimals.)*

Let us take the binary equivalent of f from the conversion table on page 15 of the first handout.

$$f = 1111_2$$

The binary equivalent of three taken from the same table is $3 = 11_2$. Let us add two zeros in front of the ones to form a group of four digits.

$$3 = 0011_2$$

(The procedure is called padding.) Finally, let us join the groups of four bits (a bit is a short form for "binary digit") together in the proper order.

$$f3 = 1111, 0011_2$$

Problem 21 *Convert both numbers to decimals to check whether the above equality is correct.*

The trick works as well in the opposite direction.

Example 4 *Convert the number* 1010011_2 *to the hexadecimal form.*

Let us split the above number into groups of four bits using padding if needed.

$$1010011 = 0101, 0011$$

According to the conversion table, $0101_2 = 101_2 = 5_{16}$ *and* $0011_2 = 11_2 = 3_{16}$. *Writing the above hexadecimal digits in the proper order finishes the solution.*

$$1010011_2 = 53_{16}$$

Problem 22 *To check whether the above equality is correct, convert both numbers to the decimal form.*

Problem 23 *Use the new method to convert the number* 100111_2 *to the hexadecimal form.*

Once finished, convert both numbers to the decimal form to check the result.

Problem 24 *Use the new method to convert the number* $1da$ *to the binary form.*

Once finished, convert both numbers to the decimal form to check the result.

Question 1 *Eight is also a power of two. Can you guess an efficient way to go back and forth between the binaries and octals?*

Problem 25 *Use the new method to convert the number* 10101_2 *to the octal form.*

Once finished, convert both numbers to the decimal form to check the result.

Problem 26 *Use the new method to convert the number* 753_8 *to the binary form.*

Once finished, convert both numbers to the decimal form to check the result.