Homework 4: Quadratic Inequalities

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1 Reading

Solution 1 (L3.2a)**.**
Let us go through the functions:
1) $f(x) = x \cdot |x|$. We will show that this is odd, using the central property of the absolute value $|x| = |-x|$:

$$f(-x) = -x \cdot |-x| = -x \cdot |x| = -f(x)$$

2) $f(x) = |x+1| - |x-1|$. This is also odd:

$$f(-x) = |-x+1| - |-x-1| = |x-1| - |x+1| = -f(x)$$

3) $f(x) = |x+1| + |x-1|$. This is even:

$$f(-x) = |-x+1| + |-x-1| = |x-1| + |x+1| = f(x)$$

4) $f(x) = 3x - x^2$. This is neither odd nor even. Indeed, $f(1) = 2$ and $f(-1) = -4$, which means that $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ at least at $x = 1$.

Solution 2 (H3.2)**.**
Let I be the intersection of diagonals of $ABCD$, which is incidentally also the center of the inscribed circle. Let E be the point at which the inscribed circle is tangent to AB, and F – the point of tangency to AD. Then note that $AE = AF$ and $IE = IF$. Then $\triangle AEI = \triangle AFI$, which in turn implies $\angle BAI = \angle DAI$. Similarly, we can show that $\angle BCI = \angle DCI$. The two angle equalities imply that $\triangle ABC = \triangle ADC$, which means that $AB = CD$ and $AD = BC$. Now we can remember that $AB + CD = AD + BC$, and we get $AB = CD = AD = BC$ which means that $ABCD$ is a rhombus.

2 Homework

Problem 1.
Show that out of all rectangles with a fixed perimeter, the square has the largest area.

Problem 2.
The circle with center O is inscribed in the quadrilateral $ABCD$. Show that $\angle AOB + \angle COD = 180°$.