Homework 3

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1 Reading

Solution 1 (L2.4)**.**
Let M be the midpoint of AC. If M lies on the circle with diameter AB, then $\angle AMB = 90°$. Then AM is the altitude of $\triangle ABC$. But it is also the median, which implies that $BA = BC$.

Solution 2 (H2.1)**.**
By Vieta's theorem, $2p$ and $p+q$ being roots of the quadratic equation $x^2 + px + q$ is equivalent to the following system of equations:

$$2p + (p+q) = -p \tag{1}$$

$$2p(p+q) = q \tag{2}$$

From the first equation we get $q = -4p$. Plugging this into the second equations yields $2p \cdot (-3p) = -4p$, which means $p = 2/3$ or $p = 0$. The latter case would imply $q = 0$, which is not allowed as $p \neq q$. Then $p = 2/3$, and so $q = -8/3$.

2 Homework

Problem 1.
Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x^2) = 4x^2 + 4x + 1$.

Problem 2.
Let $ABCD$ be a circumscribed quadrilateral. Show that if the center of the circle inscribed in $ABCD$ coincides with the intersection of the diagonals of $ABCD$, then $ABCD$ is a rhombus.