# UCLA Match Circle
October 14, 2018

1. Take a square and a rectangular card. How many symmetries do these cards have if you can flip the rectangular card, but can only rotate the square card? You can move them around, but when you are done with a move it must fit back exactly where it started. Come up with names for the moves you can do on these cards, and come up with a table listing what happens when you first do one move and then another.

   Is it possible to label the moves you can do on the two cards so that the two tables you get are the same?
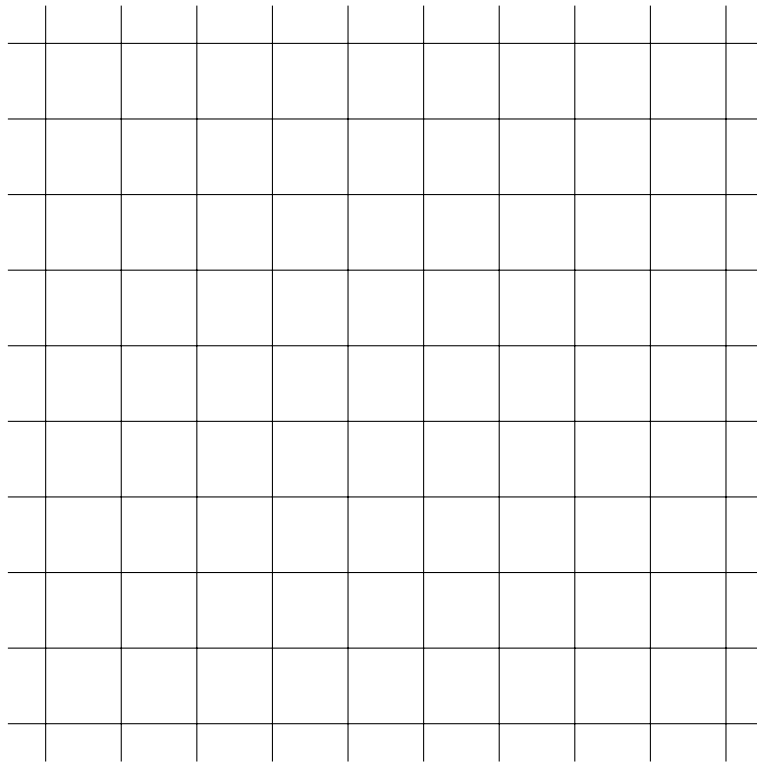
2. Now with a partner take a stick and three strings, and poke three holes in your cards, so that you now have a card attached to a stick. How many symmetries do the cards have now? You are now allowed to flip the rectangular card and rotate the square card, but two moves are the same if the card and the strings look the same. You are allowed to hold the card fixed and move it around in space (without rotating or flipping) in order to simplify the strings.

   As before, come up with names you can do on the cards, and come up with a table listing what happens when you first do one move and then another.

   Could you label these moves so that the two sets of moves are the same?

3. Now take the square, and look at symmetries where you can flip the square as well as rotate it. Make a multiplication table Is this set of symmetries that same as any of the ones coming from cards with strings attached?

4. Take an octagon, and look at symmetries where you can only rotate the octagon. Is this the same as any previously encountered symmetries?

5. Take a nickel and quarter, placed where the nickel is on the left and the quarter is on the right. Two configurations are the same if the same sides of the coins are showing, and the coins are in the same place. How many symmetries are there? Have you seen these symmetries before? Make a multiplication table showing what happens when you do move move and then another. Does the order in which you do the moves matter?

6. Now you can swap the positions of the two coins. How many moves are there? Examples of moves are things like: flip the left hand coin, flip the right hand coin, swap the two coins and then flip the left hand coin. Make a multiplication table showing what happens when you first do one move and then another. Does the order in which you do the moves matter? Have you seen this group of symmetries before?

7. We found several different things with 8 different symmetries, or 8 different moves we could do on an object and have the object look the same in the end (for example, rotating an octagon).

   When we wrote out multiplication tables for these moves, we learned that for some of the things we could rename the moves so that the multiplication tables for two different objects were the same.

   Can you come up with anything that has 8 different symmetries so that the resulting multiplication table isn't the same as one we have already seen?