Leap Frog

Los Angeles Math Circle

January 15, 2017

Game Instructions

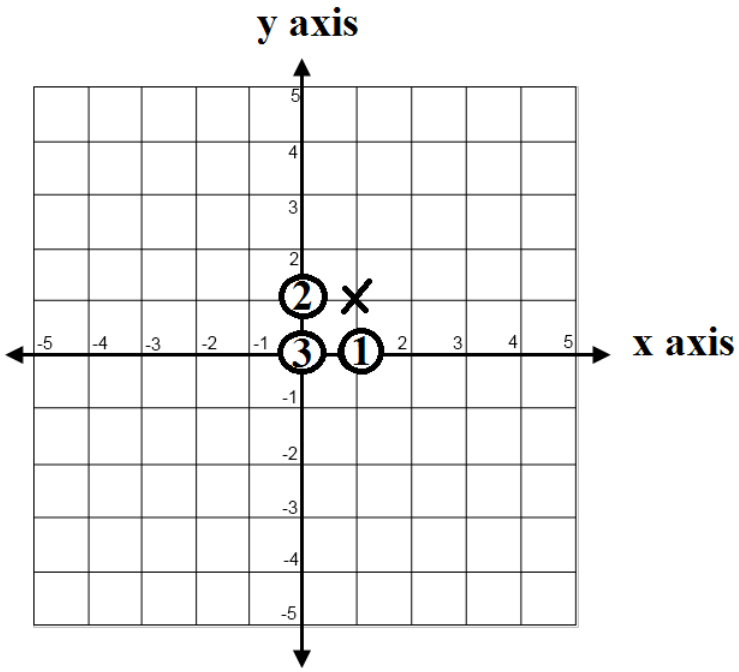
How To Start Leap Frog is a three player game. Pick a unit square on the grid. Each player puts his or his or her piece on a different corner of the square. On the corner that remains free, draw an X.

How To Play On every move, a player must "leap frog" over any other player by taking his or her piece and *reflecting* it over their opponent's piece. In other words, if it is Player 1's turn, Player 1 must move his or her piece so that some other player's piece sits in the middle between Player 1's old and new position.

How To Win The first player to land on the X drawn at the beginning of the game is the winner.

Starting Analysis

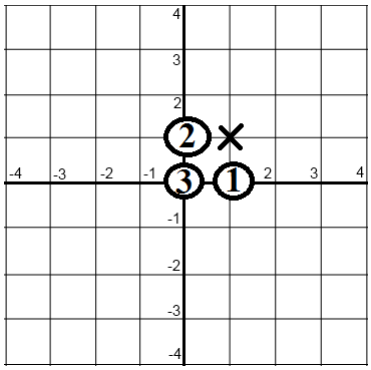
Below is a game of Leap Frog that just started.



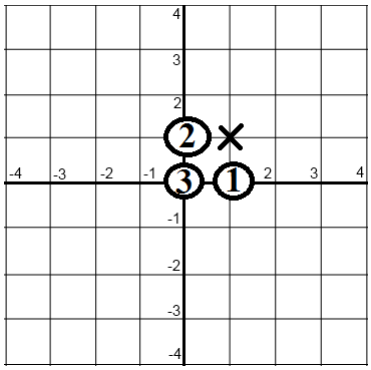
1. Fill in the position of the three players at the start of the game:

Player	x-coordinate	y-coordinate	Position
1. ①	1	0	(1,0)
2. ②			
3. ③			

2. Draw in all the spots that Player 1 can move to. Then write down what those positions are.

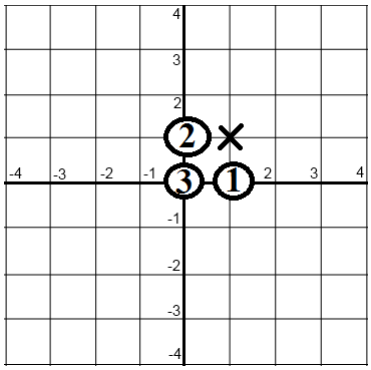


①	Position
Old	(1, 0)
New	

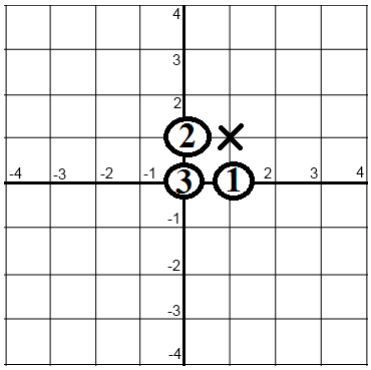


①	Position
Old	(1, 0)
New	

3. Do the same for Player 2.

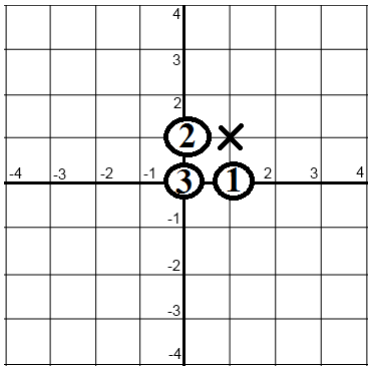


②	Position
Old	(0, 1)
New	

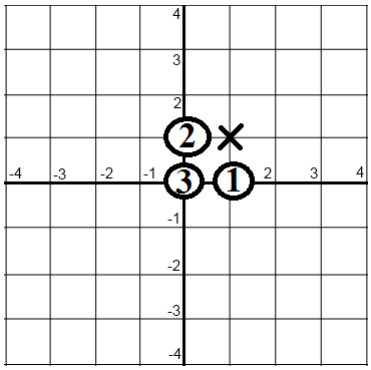


②	Position
Old	(0, 1)
New	

4. Do the same for Player 3.



③	Position
Old	(0, 0)
New	



③	Position
Old	(0, 1)
New	

Your Turn to Play

1. Play Leapfrog with two classmates at your table. Use the table below to record where each player lands on every turn.

	Player 1	Player 2	Player 3
START	(1,0)	(0,1)	(0,0)
Turn 1			
Turn 2			
Turn 3			
Turn 4			
Turn 5			
Turn 6			
Turn 7			
Turn 8			

2. Questions about Player 1.

   (a) What do you notice about Player 1's x-coordinates?

   (b) What do you notice about Player 1's y-coordinates?

(c) Is it possible for Player 1 to land on the position $(1, 1)$? Why or why not?

3. Questions about Player 2.

(a) What do you notice about Player 2's x-coordinates?

(b) What do you notice about Player 2's y-coordinates?

(c) Is it possible for Player 2 to land on the position $(1, 1)$? Why or why not?

4. Questions about Player 3.

(a) What do you notice about Player 3's x-coordinates?

(b) What do you notice about Player 3's y-coordinates?

(c) Is it possible for Player 3 to land on the position $(1, 1)$? Why or why not?

5. Is it possible for any of these three players to win the game?

The Game Has Changed

1. Imagine you start the game by placing your pieces on a 3-by-1 rectangle instead of a 1-by-1 square. Is it possible for any player to land on the winning position? Why or why not?

2. Imagine you start the game by placing your peices on a 2-by-2 square instead of a 1-by-1 square. Is it possible for any player to land on the winning position? Why or why not?

3. Is there any type of starting square or rectangle that guarantees the possibility of a winner? If so, give a general rule for such a square.

Logic Puzzles

1. Dustin comes from a town in Florida where each person wears either a red or blue beanie. If anyone figures out the color of her own beanie, she or he must leave the city. Miguel came to visit Dustin and at a town meeting announced, "how interesting it is to see someone wear a red beanie in this town." Determine Dustin's fate and explain why. *(Note: everyone in the town is perfectly logical and will make any valid logical deduction.)*

2. After a lot of petitioning, the UCLA math department decided to give every student a locker. There are 1000 lockers in total and they have been numbered from 1 through 1000. During recess (of course UCLA has recess!), the students decide to try an experiment. When recess is over each student will walk into the school one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with even numbers. The third student will change all of the locker doors that are multiples of 3 (change means closing lockers that are open, and opening lockers that are closed.) The n-th student will change the orientation of all locker doors that are multiples of n. After 1000 students have entered the school, how many locker doors will be open?

Homework

1. For homework, go home and teach someone how to play the game.