Leap Frog

Los Angeles Math Circle

January 15, 2017

Game Instructions

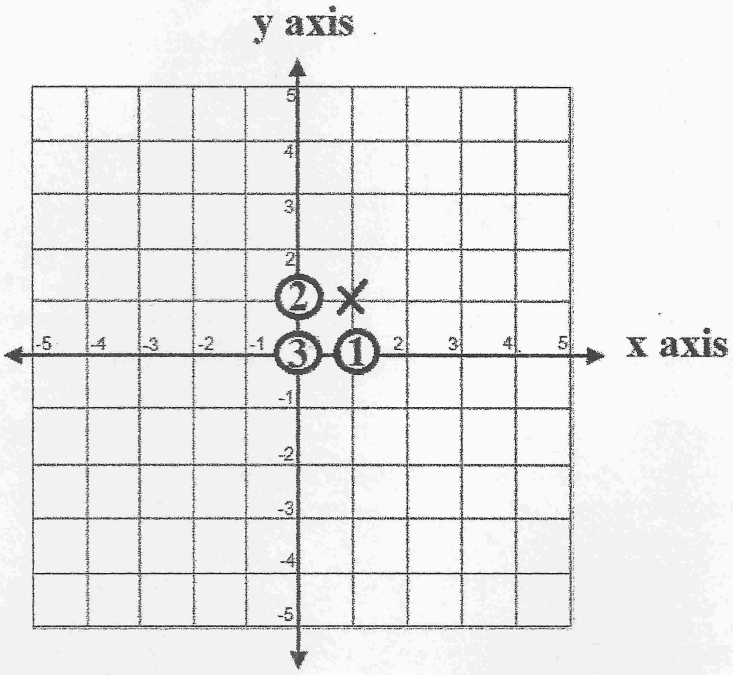
How To Start Leap Frog is a three player game. Pick a unit square on the grid. Each player puts his or her piece on a different corner of the square. On the corner that remains free, draw an X.

How To Play On every move, a player must "leap frog" over any other player by taking his or her piece and *reflecting* it over their opponent's piece. In other words, if it is Player 1's turn, Player 1 must move his or her piece so that some other player's piece sits in the middle between Player 1's old and new position.

How To Win The first player to land on the X drawn at the beginning of the game is the winner.

Starting Analysis

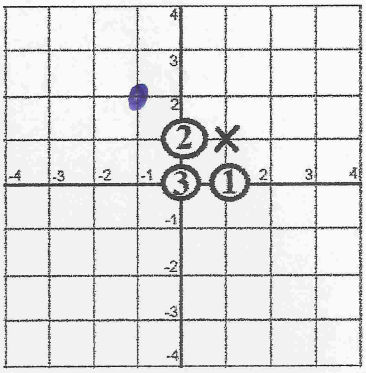
Below is a game of Leap Frog that just started.



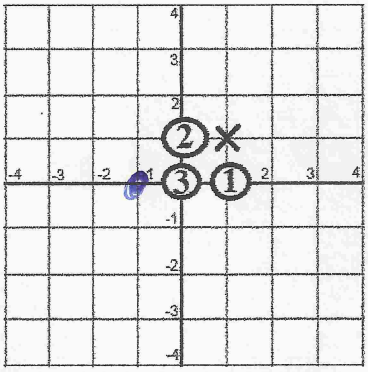
1. Fill in the position of the three players at the start of the game:

Player	x-coordinate	y-coordinate	Position
1. ①	1	0	(1,0)
2. ②	0	1	(0,1)
3. ③	0	0	(0,0)

2. Draw in all the spots that Player 1 can move to. Then write down the x- and y-coordinates of these.

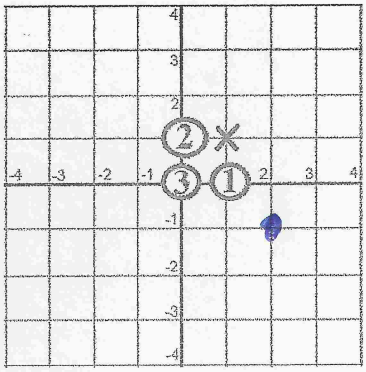


①	Position
Old	(1, 0)
New	(-1, 2)

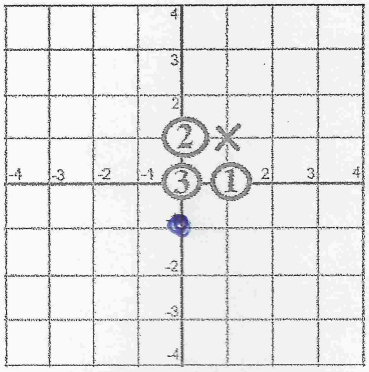


①	Position
Old	(1, 0)
New	(-1, 0)

3. Do the same for Player 2.

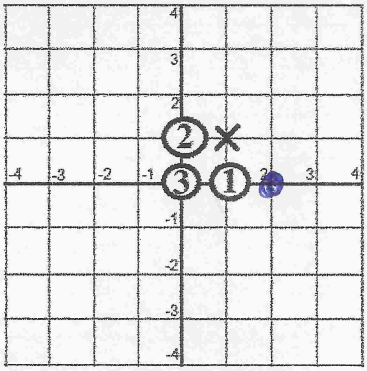


②	Position
Old	(0, 1)
New	(2, -1)

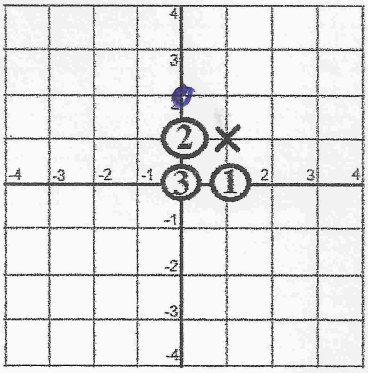


②	Position
Old	(0, 1)
New	(0, -1)

4. Do the same for Player 3.



③	Position
Old	(0, 0)
New	(1,0)



③	Position
Old	(0, 0)
New	(0,2)

Your Turn to Play

1. Play Leapfrog with two classmates at your table. Use the table below to record where each player lands on every turn.

	Player 1	Player 2	Player 3
START	(1,0)	(0,1)	(0,0)
Turn 1	(-1,0)	(0,-1)	(-2,0)
Turn 2	(1,-2)	(2,-1)	(4,-4)
Turn 3	(3,0)	(4,1)	(4,6)
Turn 4	(5,2)	(6,3)	(6,-2)
Turn 5	(7,4)	(8,5)	(8,10)
Turn 6	(9,16)	(8,15)	(8,20)
Turn 7	(7,14)	(6,13)	(4,6)
Turn 8	(1,-2)	(2,-1)	(0,-8)

2. Questions about Player 1.

   (a) What do you notice about Player 1's x-coordinates?

   They are always odd.

   (b) What do you notice about Player 1's y-coordinates?

   They are always even.

(c) Is it possible for Player 1 to land on the position $(1, 1)$? Why or why not?

No. The winning position is (odd, odd), but Player 1 will always be at position (odd, even).

3. Questions about Player 2.

(a) What do you notice about Player 2's x-coordinates?

They are always even.

(b) What do you notice about Player 2's y-coordinates?

They are always odd.

(c) Is it possible for Player 2 to land on the position $(1, 1)$? Why or why not?

No. The winning position is (odd, odd), but Player 2 will always be at position (even, odd)

4. Questions about Player 3.

(a) What do you notice about Player 3's x-coordinates?

They are always even.

(b) What do you notice about Player 3's y-coordinates?

They are always even

(c) Is it possible for Player 3 to land on the position $(1, 1)$? Why or why not?

No. The winning position is at (odd, odd), but Player 3 will always be at position

5. Is it possible for any of these three players to win the game?

No one can win the game.

The Game Has Changed

1. Imagine you start the game by placing all three players's pieces and an X on the corners of a 3-by-1 rectangle with coordinates $(0, 0)$, $(1, 0)$, $(0, 3)$, and $(1, 3)$. Is it possible for any player to land on the X? Why or why not?

No one can win. It follows the same logic as above.

2. Imagine you start the game by placing all three players's pieces and an X on the corners of a 2-by-2 square with coordinates$(0, 0)$, $(2, 0)$, $(0, 2)$, and $(2, 2)$ Is it possible for any player to land on the X? Why or why not?

No one can win. It's the same as the 1-by-1 case except this game is scaled by 2.

Logic Puzzles

1. Dustin comes from a town in Florida where each person wears either a red or blue beanie. If anyone figures out the color of her own beanie, she or he must leave the city. Miguel came to visit Dustin and at a town meeting announced, "how interesting it is to see someone wear a red beanie in this town." Determine Dustin's fate and explain why. *(Note: everyone in the town is perfectly logical and will make any valid logical deduction.)*

(Assuming "someone" means 1 person.)
Dustin must leave. He either sees one person with a red hat, and if he does not, then he must be wearing the red hat.

2. After a lot of petitioning, the UCLA math department decided to give every student a locker. There are 1000 lockers in total and they have been numbered from 1 through 1000. During recess (of course UCLA has recess!), the students decide to try an experiment. When recess is over each student will walk into the school one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with even numbers. The third student will change all of the locker doors that are multiples of 3 (e.g. 3, 6, 9...) (change means closing lockers that are open, and opening lockers that are closed.) The n-th student will change all locker doors that are multiples of n. After 1000 students have entered the school, how many locker doors will be open?

31. The number of square numbers between 1 and 1000