Russian Olympiad Problems

Math Circle (Beginner)

October 7, 2018

1. Peter exchanges stickers with his friends. For every sticker he gives someone, he gets 5 stickers back. Suppose he starts the exchange with just one sticker. How many stickers will he have after 30 exchanges?

2. Write down 7 consecutive numbers so that the digit 2 is used exactly 16 times.

3. Three jumps of a two-headed dragon equals five jumps of a three-headed dragon. It takes a two-headed dragon the same amount of time to make four jumps as it takes a three-headed dragon to make seven jumps. Which of the dragons moves faster? Explain your answer.

4. Two types of ladybugs live in the magical forest: some ladybugs have 6 dots, and the rest have 4 dots each. All the ladybugs with 6 dots always tell the truth. All the ladybugs with 4 dots always lie. You met several of these ladybugs.

   - The first ladybug told you: All of us have the same number of dots.
   - The second ladybug said: Altogether, we have 30 dots on our backs.
   - The third ladybug said: No! Altogether, we have 26 dots on our backs.

   The rest of the ladybugs each said that only one of those three ladybugs told the truth. How many ladybugs did you meet?

5. Ben multiplied a number by 10 and got a prime number. Peter multiplied the same number by 15 and also got a prime number. Could it be that both of them did their computations correctly? Explain your answer.

6. Solve the following riddle:

Here is a riddle written on a cup:
Eh is four times as much as **Oi**,
Oh is four times as little as **Ai**,
What do you get if you add all four of them up?

7. A dog and a cat are pulling a sausage in two different directions. If the dog takes a bite and run away, the cat will get 300 gr more than the dog. If the cat takes a bite and runs away, the dog will get 500 gr more than the cat. How much of the sausage will be left if each of them takes a bite and runs away?

8. Thirteen children were sitting around the table. All of the girls agreed that they will only tell the truth to each other and will lie to the boys. All of the boys agreed that they will only tell the truth to each other and lie to the girls. One of the children said to his/her neighbor on the right: The majority of us are boys. The neighbor told his/her neighbor on the right: The majority of us are girls, and so on, with the last child telling the first one: The majority of us are boys. How many boys were there at the table?

9. The Big Island and The Small Island are both rectangular in shape and are divided into several rectangular counties. Each county has a road along one of the rectangles diagonals. On each of the islands, these roads form a closed path which does not go twice through any of the points. Here is a map of the Small Island:
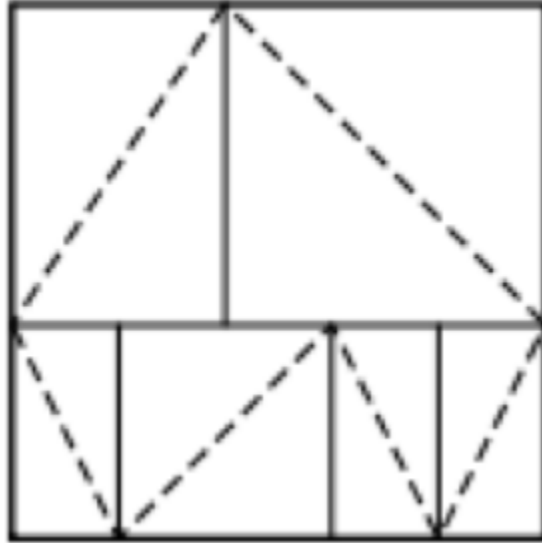

Figure 1: Small Island

Draw a possible map of the Big Island if you know that it has an odd number of counties. How many counties does your island have?

10. Thirty three giants are guarding a cave. The Wicked Witch agreed to pay them 240 gold coins under the following conditions:

- The Wicked Witch divides the giants into several troops and pays each of the troops separately;
- Within each of the troops, the coins are divided equally between the giants, and the remainder is given back to the Wicked Witch.

(a) What is the biggest number of coins that the Wicked Witch can guarantee to herself if she can give the troops different numbers of coins (Note: the total number of coins given still must be 240)?

(b) What if she has to give each troop the same number of coins (independently of how many people are in each of the troops)?

11. Put signs of mathematical operations and parentheses in such a way that you get a true statement:

$$\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad = \quad 2 \tag{1}$$

12. The following problem is attributed to Sir Isaac Newton: 70 cows eat the grass on a field in 24 days. 60 cows eat the grass on the same field in 30 days. How many cows would it take to eat all the grass in 96 days? (Hint: the grass continues to grow at a constant rate while the cows are eating it).