Lesson 9: Games and Geometry IV

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Problem 1.
Show that in a game of tic-tac-toe on an infinite board the second player does not have a winning strategy. In infinite tic-tac-toe one needs 5 in a row to win.

Problem 2.
Two players are playing a game at night on the streets of the Candy Kingdom. The streets of the Candy Kingdom make a rectangular grid. Every turn consists of finding a not yet lit intersection, and putting a projector there, which lights up everything to the right and up of itself (including the intersection it is on). The person, after whose move the whole kingdom is lit for the first time loses. Who has a winning strategy?

Problem 3.
Two players are playing a game, which starts with 3 piles of stones – one with 1, another with 2 and another with 3 stones. With each move a player can take any amount of stones from any one pile. The player who takes the last overall stone wins. Who has the winning strategy?

Problem 4.
Kiselev 253, page 96.

Problem 5.
Let two circles with centers O, O' intersect at points P and Q. Also suppose that the tangent lines to the circles at P are perpendicular. Show that points P, Q, O, O' lie on one circle.

Problem 6.
Suppose n points are marked on the plane, where $n \geq 9$. It is known that for any 9 of the points one can draw two circles so that all 9 points lie on those circles. Show that it is possible two draw two circles so that all n points lie on them.