

# Lesson 8: Discrete Finite Automata

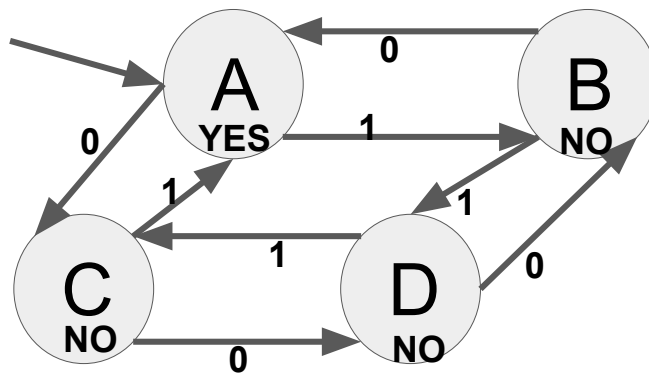
Kyle Hess

June 6, 2018

## Problem 1.

a) For each of the following strings say whether the following DFA accepts it:

- 10011
- 10010010
- 11001010001110010110100000



b) Give an intuitive description as to what set of strings this DFA accepts.

## Problem 2.

a) Write a DFA that accepts the set of binary numbers divisible by five.

b) Write a DFA that accepts the set of binary strings with the same numbers of zeroes as ones mod 3.

**Problem 3.**

a) Prove that if you run a DFA  $M$  with  $k$  states on a string  $\alpha$  of size  $n > k$  and it outputs yes, then there is a string  $\beta$  of size  $l \leq k$  such that  $M$  also outputs yes when you input  $\beta$ .

b) Prove that if you run a DFA  $M$  with  $k$  states on a string  $\alpha$  of size  $n > k$  and it outputs yes, then  $\alpha$  can be written as  $xyz$  where  $y$  is not the empty string and  $M$  outputs yes on  $xy^mz$  for any  $m$ , where  $y^m$  represents  $y$  written  $m$  times.

c) Show that there is no DFA which says yes on a string if and only if it is of the form  $0^n1^n$  for some  $n$ .

**Problem 4.**

Prove that if there is a DFA  $M$  reading left to right which accepts a certain set of strings, then there is also a DFA  $M'$  which reads right to left also accepting the same set of strings. (Note that this is the only time we allow DFA's to scan left-to-right or right-to-left; always assume I mean a left-to-right scanning DFA in the above questions).