

Lesson 6 Problem 4 Solution

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Problem 4.

a) Since $\angle HLC = \angle HKC = 90^\circ$, the quadrilateral $HKCL$ has opposing angles adding up to 180° , and thus is cyclic. Then $\angle HLK = \angle HCK$ since they intercept the same arc, and so $\angle BLK = \angle HCK$.

b) Continuing in the framework of the previous problem, we know that $\angle BLA = \angle BKA = 90^\circ$, which implies that $BKLA$ is cyclic. Then $\angle BLK = \angle BAK$, and using the result of the previous problem $\angle HCK = \angle BAK$. Let T be the intersection of line CH with BA . Then we know that

$$\angle BTC = 180^\circ - \angle ABC - \angle TCB = 180^\circ - \angle ABC - \angle BAK = 90^\circ$$

with the last equality coming from the triangle BAK . Then CH is the third altitude of $\triangle ABC$, and we are done.