

# Lesson 3 Problem 3 Solution

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## Problem 3.

We will show this by induction. Base:  $1/3 + 1/4 = 7/12 = 14/24 > 13/24$ .

Step: by the induction hypothesis,

$$\frac{1}{n+1} + \dots + \frac{1}{2n} > \frac{13}{24}$$

We want to show this when  $n$  is replaced by  $n+1$ :

$$\frac{1}{n+2} + \dots + \frac{1}{2n+2} > \frac{13}{24}$$

We will in fact show that the quantity on the left increased. Indeed, let us label

$$\frac{1}{n+1} + \dots + \frac{1}{2n} = S$$

Then

$$\begin{aligned} \frac{1}{n+2} + \dots + \frac{1}{2n+2} &= \left( \frac{1}{n+1} + \dots + \frac{1}{2n} \right) + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1} \\ &= S + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1} \\ &= S + \frac{2n+2 + 2n+1 - 2(2n+1)}{(2n+1)(2n+2)} \\ &= S + \frac{1}{(2n+1)(2n+2)} > S \end{aligned}$$

Therefore if  $S > 13/24$ , then

$$\frac{1}{n+2} + \dots + \frac{1}{2n+2} > S > \frac{13}{24}$$

which concludes the inductive step.