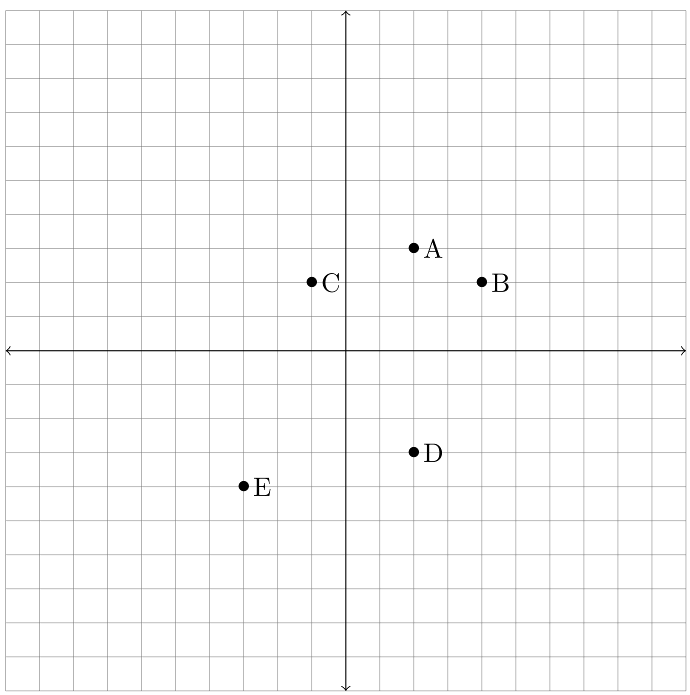
Taxicab Geometry Part II Meeting 3

Preston Carroll

22 April 2018

1. Find the taxicab distance between two consecutive letters:



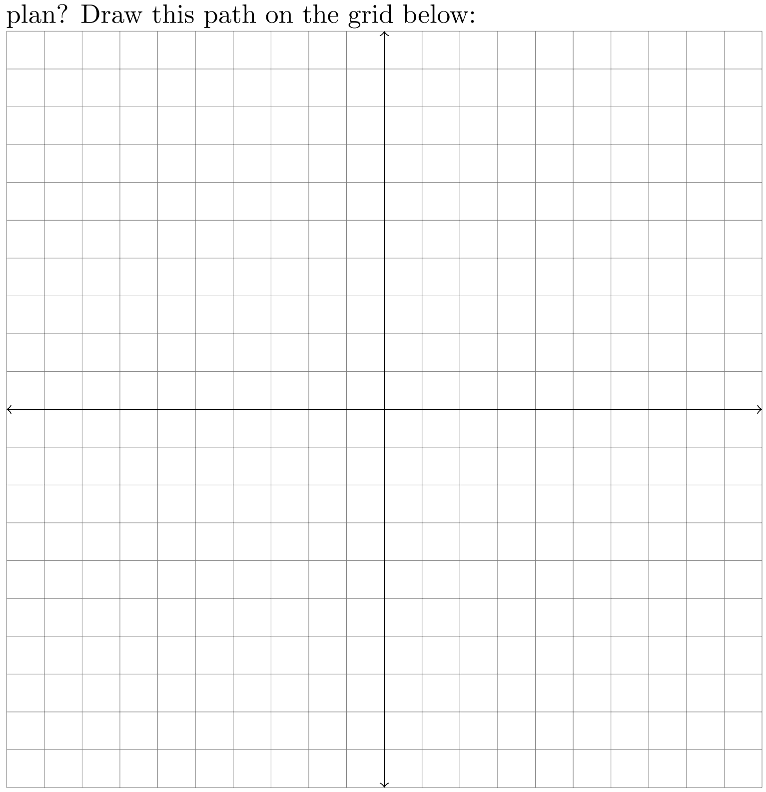
   (a) AB=
   (b) BC=
   (c) CD=
   (d) DE=

2. Bob the taxi driver's passenger wants to go from (1,1) to (6,6) as follows:

   - 4 blocks down;
   - 5 blocks to the right;
   - 5 blocks up;

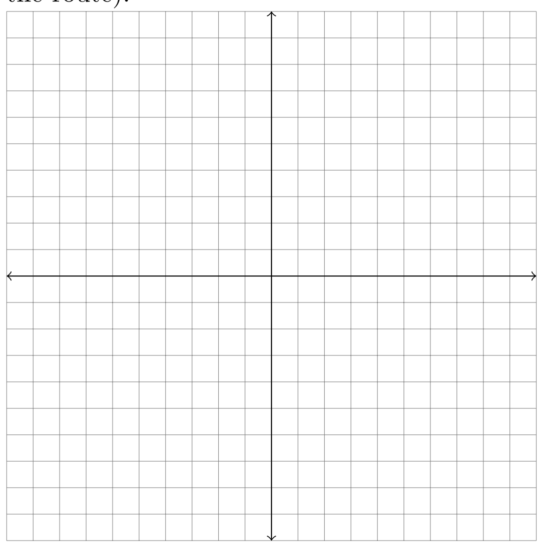
   This can be described by the following code:
   4↓, 5→, 5↑

   (a) Will Bob's passenger get to (6,6) if he starts at (1,1) and follows this plan? Draw this path on the grid below:

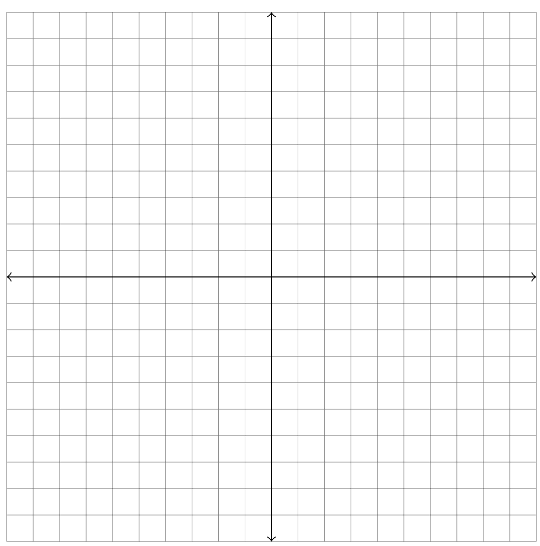
   

   (b) How many blocks in total does this route take?
   (c) Bob says that he can get from the (1,1) to (6,6) in fewer blocks. Can you draw such a route? Use a new color to draw this route if you find one.
   (d) Find length of the shortest route you can find?
   (e) What is Bob's taxicab distance from (1,1) to (6,6)

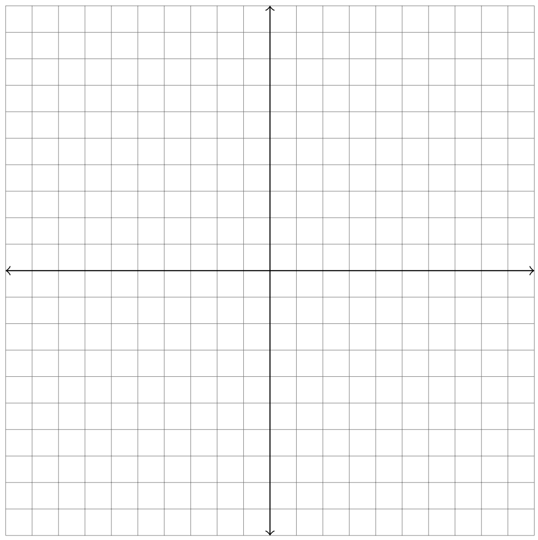
3. Find 4 possible routes starting from (0,0) that have a length of 10. Draw each of these routes and describe it by our code. (Write the code next to the route):



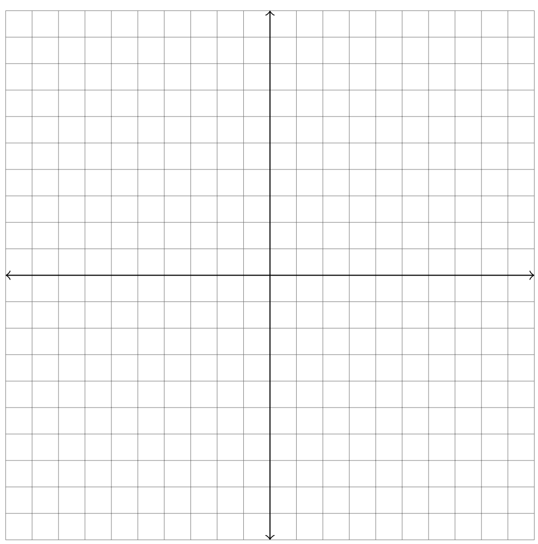
Code:



Code:

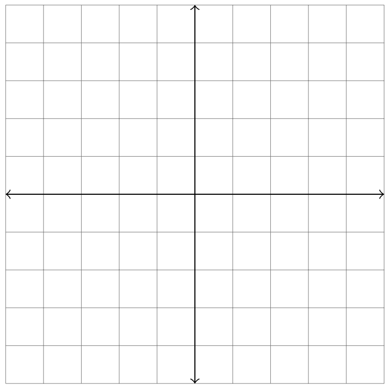


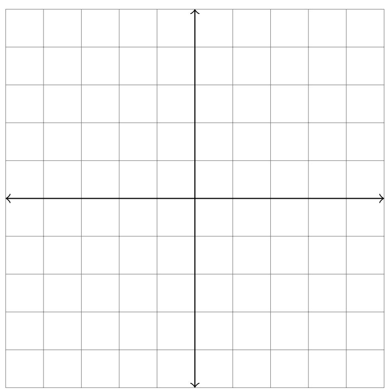
Code:

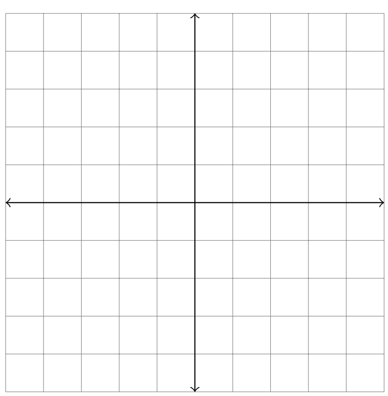


Code:

4. Find distance between the points (0,0) and (2,3) according to Bob? Draw at least two possible routes that Bob can take from (0,0) to (2,3). Do they have the same length? Remember that Bob selects the shortest routes .



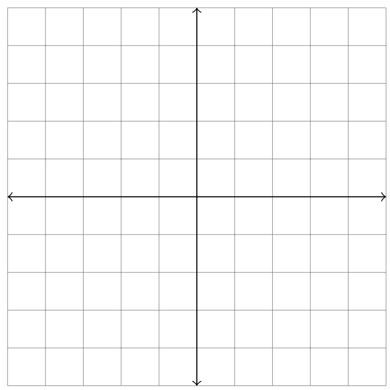




5. Bob's passenger wants to go from (0,0) to (4,5). He thinks that the distance between these two points is 11 because when he drives himself he uses the following route:
   5→, 5↑, 1←

   (a) Is he taking the shortest possible route?

   (b) Give an example of a shortest route. Does this route have a length of 11 or shorter? Draw the route and write down the code for it.

   Code:

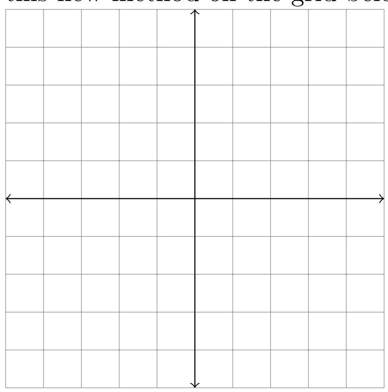
6. The mayor of the city wants to go from (0,0) to (1000,1000). The city council proposed several routes that start as follows:

   - $100\uparrow, 53\rightarrow, 27\downarrow, 50\rightarrow, 50\uparrow, ...$
   - $230\leftarrow, 60\uparrow, 80\rightarrow, 330\downarrow, 145\rightarrow, ...$
   - $500\uparrow, 200\rightarrow, 200\uparrow, 300\rightarrow, 300\uparrow, ...$
   - $400\rightarrow, 200\uparrow, 300\rightarrow, 100\downarrow, 350\rightarrow, ...$
   - $600\uparrow, 145\downarrow, 500\rightarrow, 200\leftarrow, 700\uparrow, ...$

   Assume the mayor wants to get there as fast as possible. What routes should he <u>not</u> take? Why? Cross out routes that he should <u>not</u> take.

   Select the best route of the ones listed above. What would you need to add to the code to complete the route?

7. After working for a while, Bob decided to always go horizontal by as much as necessary and then vertical by as much as necessary; this will give us a code that always consist of <u>at most</u> 2 steps. Draw the routes that follow this new method on the grid below:

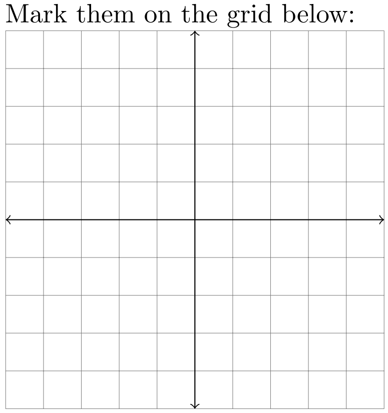


Starting Point	Ending Point	Code	Bob's Distance
(3,3)	(4,6)	$1\rightarrow$, $3\uparrow$	4
(4,4)	(-1,2)		
(2,1)	(5,3)		
(5,3)	(1,1)		
(7,5)	(1,0)		
(0,0)	(a,b)		

(a) When does this code have only one step?

(b) If Bob's route goes horizontally and then vertically, how is Bob's distance expressed in terms of the numbers in the code? Give an example and a general explanation.

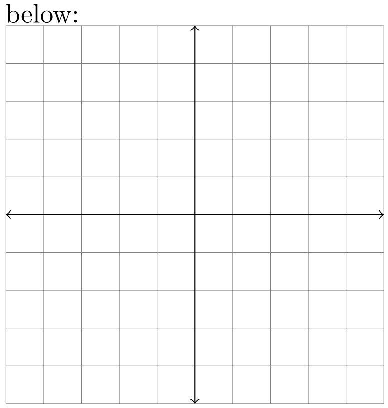
8. Bob starts at the origin and goes at the speed of one block per minute. This means that it takes him 4 minute to go from (0,0) to (4,0). This also means that it takes him 4 minutes to go from (0,0) to (0,4). What other points can he reach in 4 minutes if he continues to start at the origin? Mark them on the grid below:



How many of these points are there?

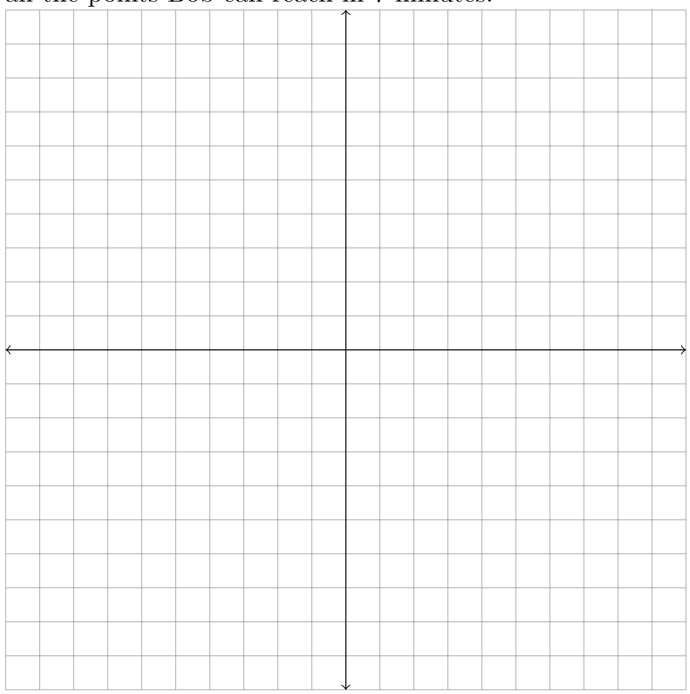
Connect these points with a ruler. What shape did you just create?

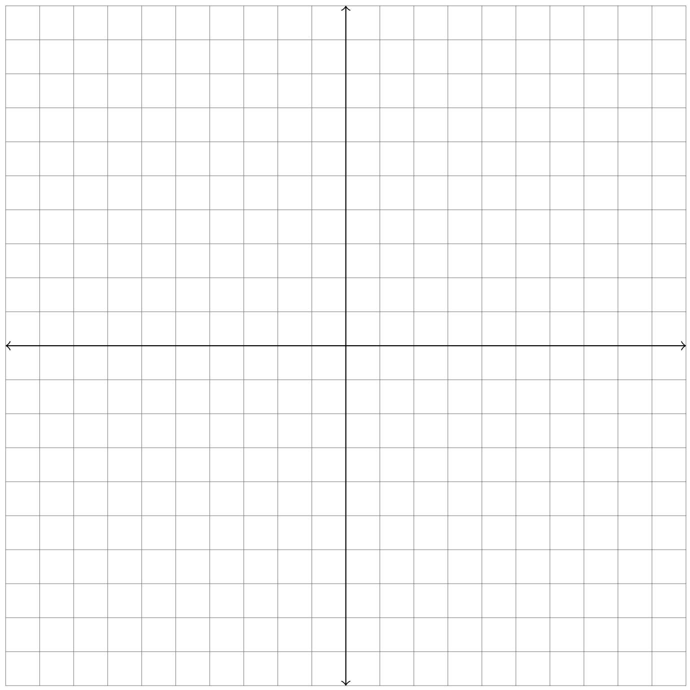
9. Now repeat this procedure except find all the points Bob can reach in exactly 5 minutes. Assume Bob is always traveling away from the origin (he doesn't turn around and waste gas). Mark all such points on the graph below:



Connect these points with a ruler. What shape have you created?

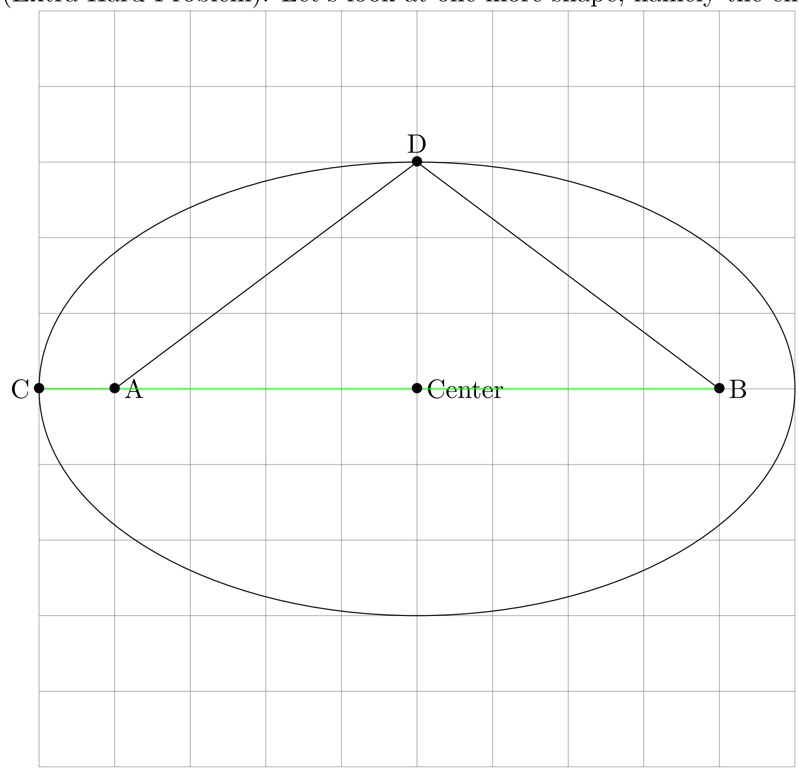
10. Try this procedure two more times with Bob being able to drive for some number of minutes strictly greater than 5. For example, you could find all the points Bob can reach in 7 minutes.





In each of these cases what shape did you find? This is what we call a Taxicab circle because it satisfies our definition of a circle, but requires we use Taxicab distances.

11. (Extra Hard Problem): Let's look at one more shape, namely the ellipse:



For all of the below find the <u>normal</u> distance (not the taxicab distance).
Hint: Use the Pythagorean Theorem ($a^2 + b^2 = c^2$)

Find the length of AD:

Find the length of BD:

What is AD+BD?

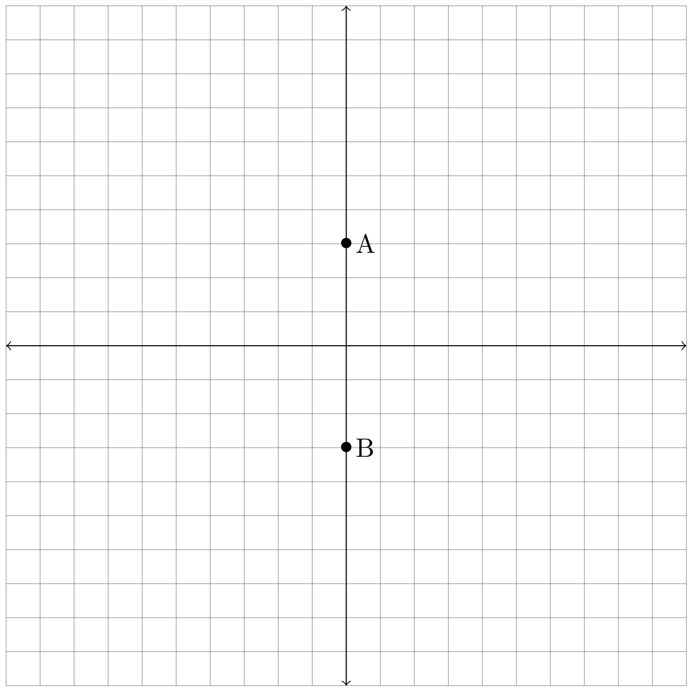
Find the length of AC:

Find the length of BC:

What is AC+BC?

This is how we define an ellipse. We take two points (Points A and B) and find all of the points whose sum of their distances to those two points are the same. In this case, we found all of the points for which the sum of the distances to A and B is 10.

Try applying our idea of taxicab distances to draw a taxicab ellipse. On the next page draw me two different taxicab ellipses. Use this page to sketch some ideas before using the grids. Only put your final answer on the next page. Try using (0,3) and (0,-3) as points A and B for your first ellipse.


A
B