

Lesson 1 Problem 3 Solution

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Problem 3

We'll prove by induction on n . Base case: when $n = 1$, the claim becomes "1+2 is divisible by 3", clearly true. Suppose, as induction hypothesis, that $n^3 + 2n$ is divisible by 3. We want to show that $(n + 1)^3 + 2(n + 1)$ is divisible by 3.

$$\begin{aligned}(n + 1)^3 + 2(n + 1) &= n^3 + 3n^2 + 3n + 1 + 2(n + 1) \\ &= (n^3 + 2n) + 3(n^2 + n)\end{aligned}$$

The first part is divisible by 3 by induction hypothesis and the second part is clearly divisible by 3. So the right hand side is also divisible by 3, completing induction.