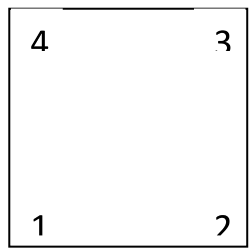
Junior Circle Meeting 9 – Commutativity and Inverses

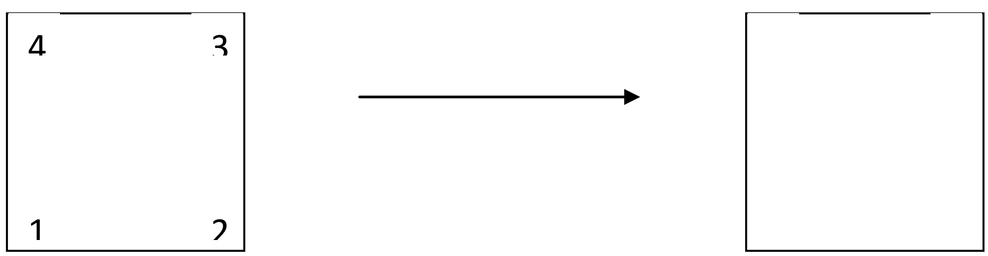
May 30, 2010

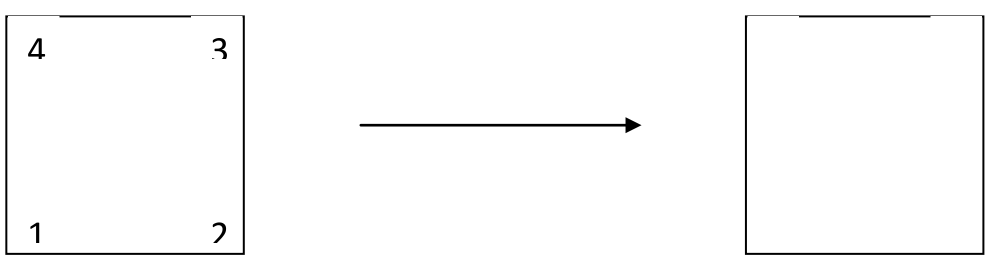
We are going to examine different ways to transform the square below:



Just as with the triangle from last week, we are going to examine flips and rotations.

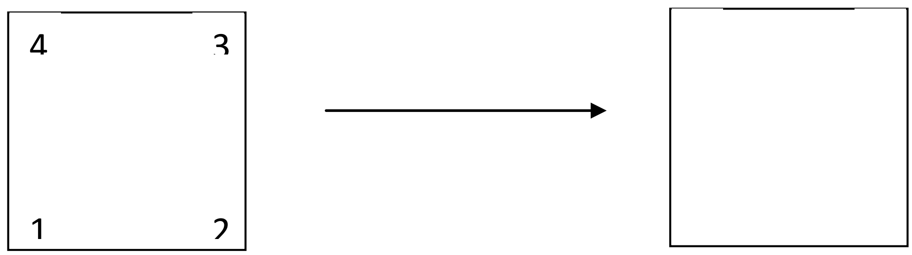
1. There are four ways to flip the square.
   a. The first two kinds of flips have the axis of symmetry through two opposite vertices. In the squares below, draw the axes of symmetry and the results of each of the flips:

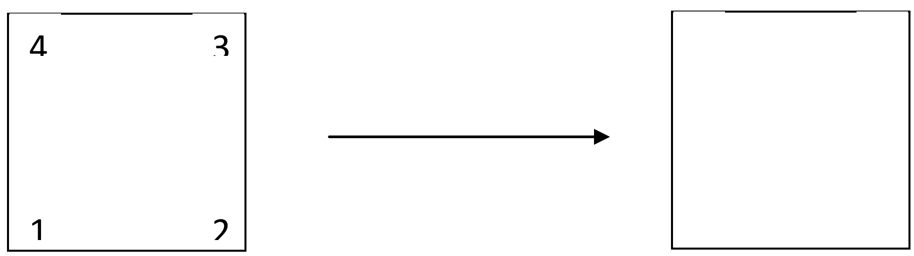




Let's call the flips through the vertices F_{13} and F_{24} because the axis of symmetry goes through vertices 1 and 3 for the first flip and vertices 2 and 4 for the second flip.

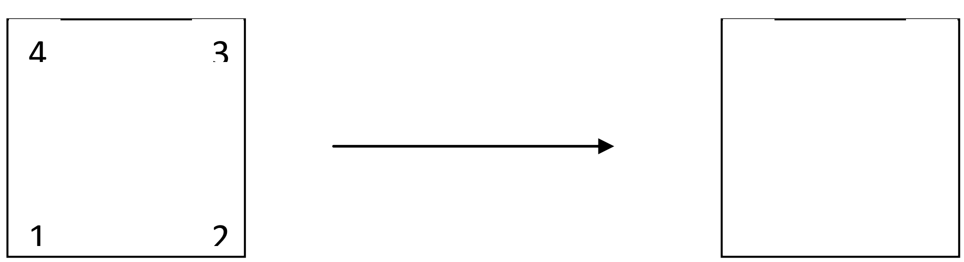
b. The second two kinds of flips have an axis of symmetry that does not going through any of the vertices. In the squares below, draw the axes of symmetry and the results of each of the flips:



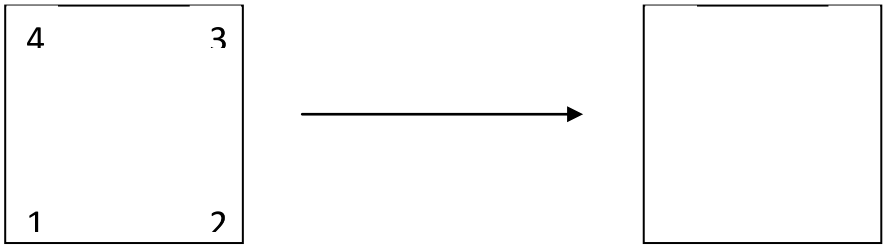


Let's call these flips F_h and F_v, where F_h means we flip over the *horizontal* axis of symmetry and F_v means we flip over the *vertical* axis of symmetry.

2. Now let's rotate our square.
   a. Draw the result of rotating the square one quarter turn in the clockwise direction:

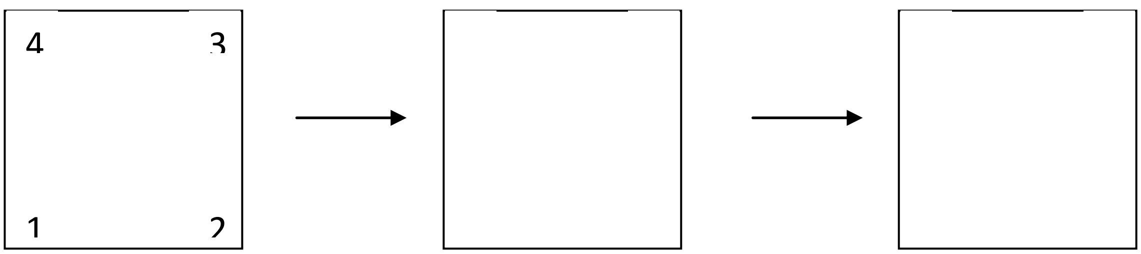


b. Draw the result of rotating the square one quarter turn in the counterclockwise direction:

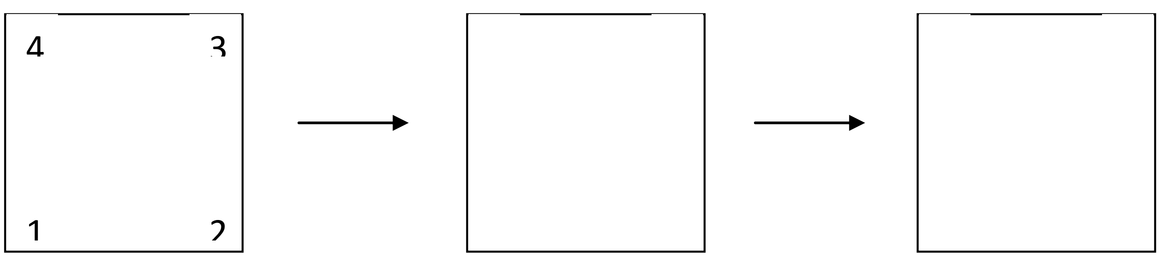


We will call these $\overset{\curvearrowright}{R}$ for clockwise rotation and $\overset{\curvearrowleft}{R}$ for counterclockwise rotation, just like we did last week.

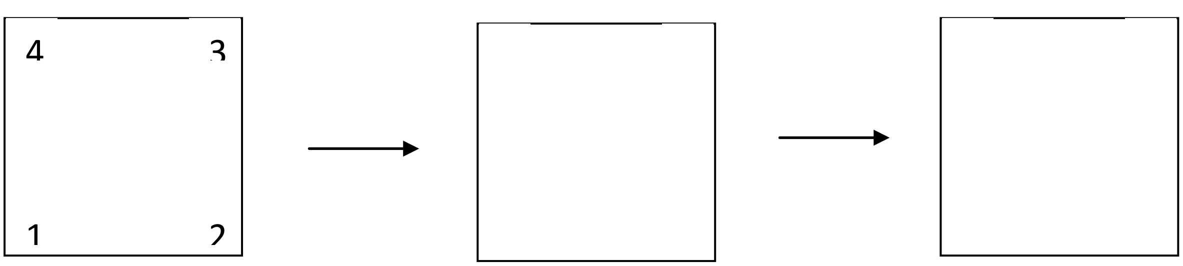
c. What happens if we perform $\overset{\curvearrowright}{R}\ \overset{\curvearrowright}{R}$? Draw the result below.



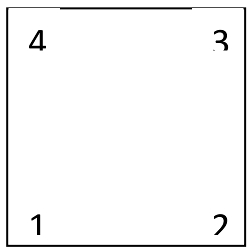
d. What happens if we perform $\overset{\curvearrowleft}{R}\ \overset{\curvearrowleft}{R}$? Draw the result below.



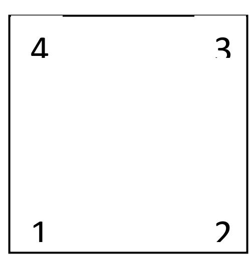
e. What happens if we perform $\overset{\curvearrowright}{R}\ \overset{\curvearrowleft}{R}$? Draw the result below.



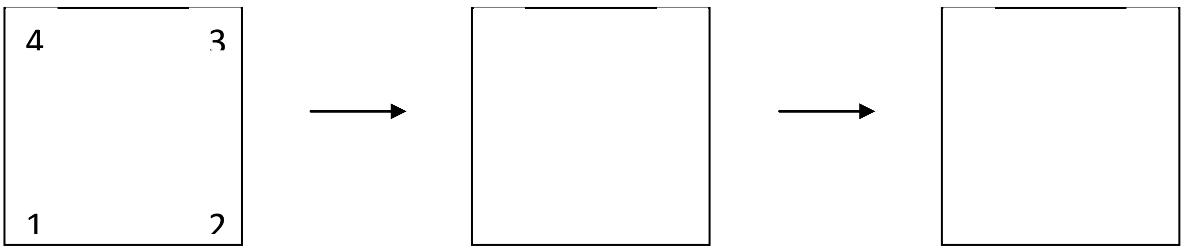
3. There are two other ways to transform our square!
    a. We can translate, or “slide” our square.   Draw the result of translating our square three inches to the right (approximately).

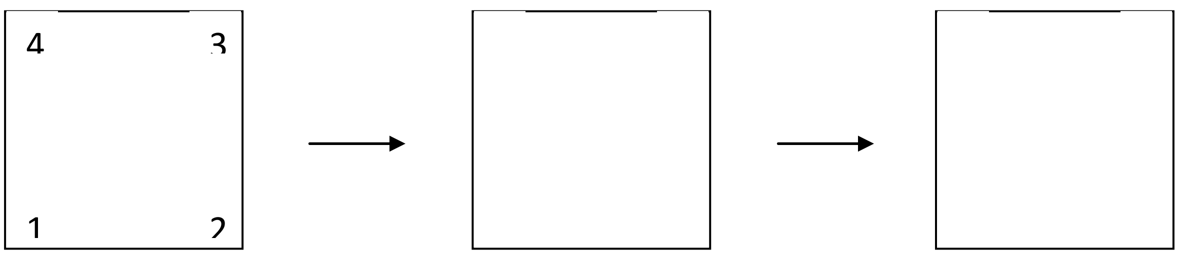


    b. We can magnify or shrink our square. Draw the resulting square if we double each of the sides. Note: you can draw the smaller square in the center of the larger square, or you could also draw the smaller square in the corner or the larger square. Either interpretation works!



4. Now we want to see which of these transformations commute. Remember: A pair of transformations commutes if we can reverse the order of the transformations and still get the same result!
    a. Diagonal flipping and Rotating: Pick a diagonal flip (F_{13} or F_{24}) and pick a rotation (R or R) and see if they commute!

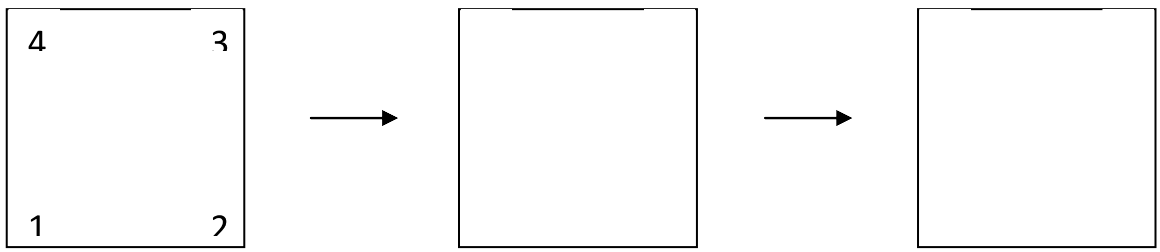


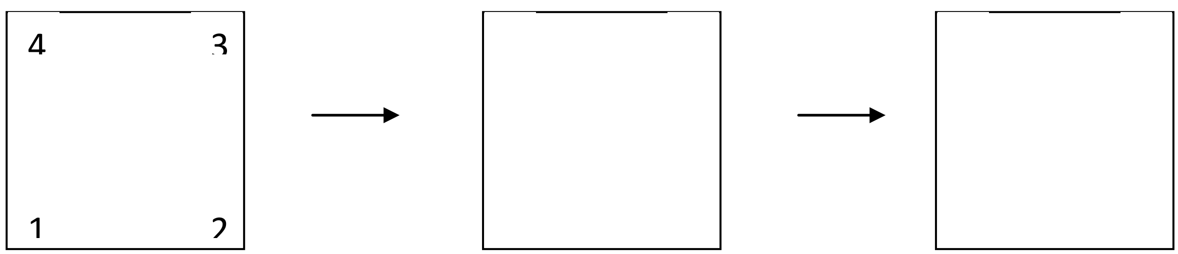


Did they commute?

How does this compare to your results with the triangle from last week?

b. Horizontal/Vertical Flipping and Rotating: Pick the horizontal or vertical flip (F_h or F_v) and pick a rotation (R or R) and see if they commute!

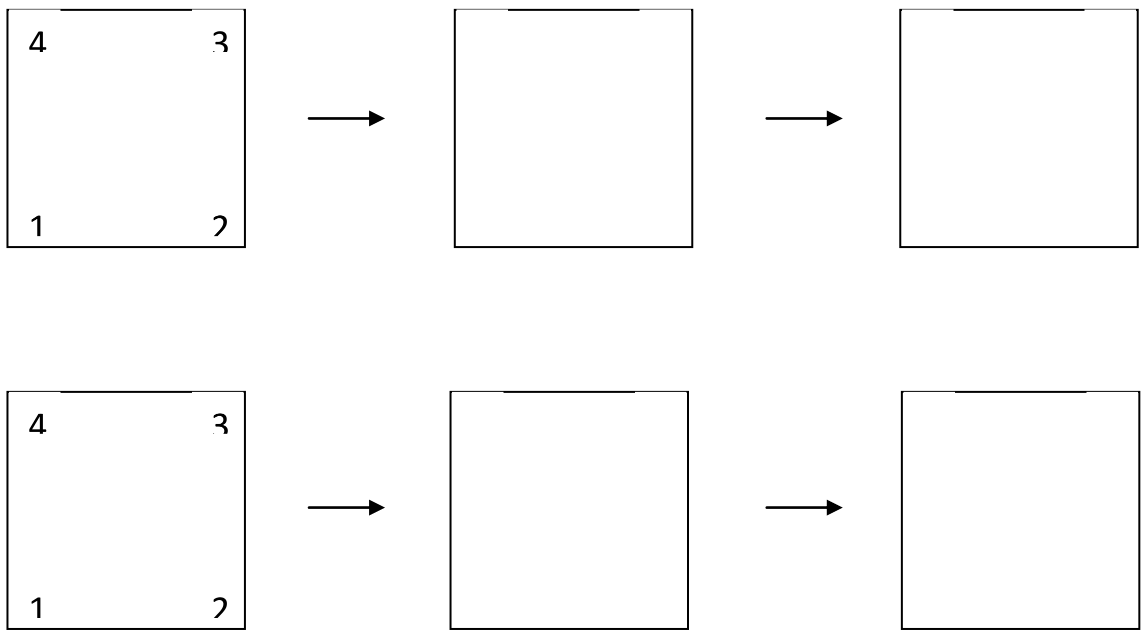




Did they commute?

How does this compare to your results with the triangle from last week?

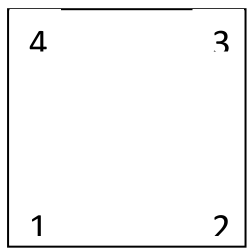
c. Flipping and Flipping: Pick two flips from F_{13}, F_{24}, F_h, or F_v and see if they commute!



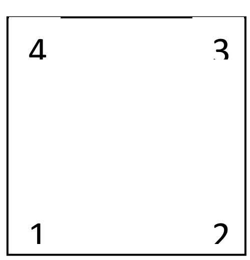
Did they commute?

How does this compare to your results with the triangle from last week?

d. Translating and Magnifying: First translate the square three inches to the right, then double the sides.



Now double the sides, and then translate the square three inches to the right.



Did they commute?

5. Can you find two mathematical operations, other than the geometric transformations that we have already examined, that do not commute? Explain why your operations do not commute.

6. We have examined whether or not certain mathematical operations commute. We can also examine whether a single operation commutes on the numbers it takes as input! We call an operation on two numbers *commutative* if we can switch the order of the numbers and still get the same result!

   For instance, addition is commutative since, for example, 1+2 equals 2+1.

   On the other hand, an operation that takes two numbers and makes the first number the numerator and the second number the denominator of a fraction is not commutative. For example, $\frac{1}{2}$ does not equal $\frac{2}{1}$.

   For each of the operations below, state whether or not the operation is *commutative*. If it is not, provide a *counterexample*.

   a. Multiplication

   b. Subtraction

   c. Division

d. Finding the maximum of two numbers, a and b.

e. Finding the minimum of two numbers, a and b.

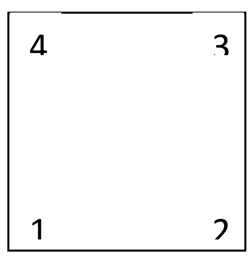
7. We now examine the *inverse* of an operation. If we apply an operation, and then apply the inverse, we get back what we started with!

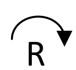
   For example: if the operation is "adding 1 to a given number", then the inverse is "subtracting 1 from a given number". For example,

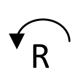
   5 + 1 = 6 and

   6 -1 = 5.

   Let's examine what this means for the transformations we found for our square:



a. What is the inverse of ?

b. What is the inverse of ?

c. What is the inverse of F_{13}?

d. What is the inverse of F_{24}?

e. What is the inverse of F_h?

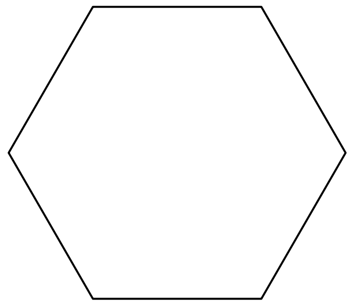
f. What is the inverse of F_v?

g. What is the inverse of translating 3 inches to the right?

h. What is the inverse of doubling the sides of the square?

8. Not all operations have inverses. Can you think of some examples in everyday life that can only go “forwards” but not “backwards”?

9. Now let’s examine the ways to transform the hexagon below:



a. Label the six vertices with the numbers 1 through 6 in the counterclockwise direction (as done with the triangle and square).

b. How many different flips are there? Draw an axis of symmetry for each of the flips.

c. What are the possible rotations?

d. Give an example of a pair of transformations of the hexagon that commute.

e. Give an example of a pair of transformations of the hexagon that do not commute.

10. Let’s now look for a pattern with the number of flips.
    a. How many flips are there for the triangle?

    b. How many flips are there for the square?

    c. How many flips are there for the hexagon?

    d. If we have a polygon with 100 equal sides and equal angles, how many different flips will there be?

    e. How many possible flips are there for a circle? Why?

11. A *palindrome* is a word, phrase, or number that can be read the same way in either direction. In a way, it has an axis of symmetry!
    a. Write down a number that is a palindrome.

    b. Can a palindrome number be any length of digits? (For example, 2-digit, 3-digit, etc).

    c. Can you find some words that are palindromes?

    d. Can you think of a phrase that is a palindrome?