Lesson 6 Problem 5 Solution

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March 9, 2018

Problem 5.

Let the parallel chords be AB and CD. We want to show that $\overset{\frown}{AC}=\overset{\frown}{BD}$. Let O be the center of the circle. Consider the diameter perpendicular to AB and CD. Let it intersect AB at M and CD at N. Then we have 2 cases: if O is between AB and CD, then $\angle AOM = \angle BOM$ and $\angle CON = \angle DON$. Then

$$\overset{\frown}{AC}= \angle AOC = 180^\circ - \angle AOM - \angle CON = 180^\circ - \angle BOM - \angle DON = \angle BOD = \overset{\frown}{BD}$$

and we are done. The second case is when AB and CD are on the same side of O. Without loss of generality we can assume that AB is closer to O. Then

$$\overset{\frown}{AC}= \angle AOC = \angle AOM - \angle CON = \angle BOM - \angle DON = \angle BOD = \overset{\frown}{BD}$$

are we are still done.