How to Fight a Dragon*

February 25th, 2018

Warm up problems

1. In a Round Robin chess tournament every two participants play exactly one game with every other participant in the tournament. Three math circle students organized a Round Robin tournament. After each game, the winner of this game gets 2 color pencils (and the other person gets none). If a game ends in a draw, each player gets a color pencil. How many color pencils will be distributed in this tournamnet?

2. Put $+$ or $-$ sign between *some* of the digits to get a correct statement. (Note that there might be no sign between some of the digits):

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad = \quad 10$$

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad = \quad 38$$

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad = \quad 78$$

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*The idea of the problem is borrowed from the textbook "Mathematics" by L.G. Peterson (in Russian)

3. Parker picked a number. She added three to the number. Then she multipled the number by 2. Then she subtracted 3. She ended with 19.What number did Parker pick at the beginning?

4. Dashel decides that he is going to give his neighbors oranges from his tree. However, the oranges are very heavy so Dashel wants to get rid of them as fast as possible. He gets to Mrs. Robinson's house first and gives her half of his oranges. He goes to Mr. Smith's house next and gives him half of the oranges he has left. Lastly, he goes to Mrs. Scott's house and gives her half of the rest of his oranges. As he walks back to his house, he looks in his bucket and sees that he has 6 oranges left. How many oranges did Dashel pick off of his tree at the beginning of the day?

Far away, in a Magic Land, there are a lot of dragons and a lot of heroes. Each of the dragons has some number of Heads (H) and some number of tails (T). To defeat a dragon, a hero needs to get rid of all of its heads and all of its tails.

Here are the rules of the fight. In a single blow, a hero can cut off 1 or 2 heads, or 1 or 2 tails. A hero can not cut off a head and a tail at the same time. Moreover,

- If you cut off 1 Head, then 2 Heads immediately grow instead;

- If you cut off 2 Heads, nothing grows back;

- If you cut off 1 Tail, then 2 Tails immediately grow back;

- If you cut off 2 Tails, then 1 Head immediately grows back.

Today we will come up with a plan of defeating a dragon with a specific number of heads and tails. A dragon with H heads and T tails will be denoted by (H, T). For example, suppose you start fighting with a dragon that has 3 heads and 5 tails. Suppose you first cut off 1 head. Then, 2 heads grow instead. As a results, the dragon now has 4 heads and 5 tails. We will write this as follows:

$$(3, 5) \xrightarrow{1H} (4, 5)$$

Let's Practice! Please write the correct form for the following dragons. (H,T)

- Stella the Dragon has 2 Heads and 4 Tails

- Stacy the Dragon has 1 Tail and 3 Heads

- Spencer the Dragon has 1 Head and 5 Tails

1. Fight the following dragons and write down what you get as a result:

   - Cut 1 head from the following dragons:

     – $(1, 5) \longrightarrow$

     – $(2, 4) \longrightarrow$

   - Cut 2 heads from the following dragons:

     – $(2, 2) \longrightarrow$

     – $(3, 1) \longrightarrow$

   - Cut 1 tails from the following dragons:

     – $(1, 5) \longrightarrow$

     – $(2, 4) \longrightarrow$

   - Cut 2 tails from the following dragons:

     – $(1, 5) \longrightarrow$

     – $(2, 4) \longrightarrow$

2. For the $(3, 3)$ dragon, complete all the 4 types of moves that are possible.

$$(3, 3) \xrightarrow{1H}$$

$$(3, 3) \xrightarrow{2H}$$

$$(3, 3) \xrightarrow{1T}$$

$$(3, 3) \xrightarrow{2T}$$

3. A hero is fighting with a dragon. You see the dragon in the beginning and after the first hit. Can you determine what the hero cut off in this turn? Write the answer above the arrow.

- $(2,3) \longrightarrow (3,3)$;

- $(5,1) \longrightarrow (5,2)$;

- $(3,5) \longrightarrow (4,3)$;

- $(2,4) \longrightarrow (2,5)$;

4. We can record the whole fight in this way! For example, here is a way to defeat the $(3,0)$ dragon. Describe what happens at each step in the fight (find the number of heads (H) and tails (T) that were cut off and write it above the arrow). The first step is done for you.

$$(3,0)\xrightarrow{1H}(4,0)\longrightarrow(2,0)\longrightarrow(0,0).$$

5. Come up with a plan to defeat the following dragons as quickly as possible:

- $(1, 0) \longrightarrow$

- $(4, 1) \longrightarrow$

- $(3, 2) \longrightarrow$

- $(1, 1) \longrightarrow$

6. The biggest dragon in the Magic Land has 2018 heads but no tails. Explain how you can defeat this dragon.

$$(2018, \ 0) \longrightarrow$$

7. Another huge dragon has 1 head and 2018 tails. Explain how you can defeat this dragon.

$$(1, \ 2018) \longrightarrow$$

8. Explain how you can defeat a dragon with an even number of heads and no tail.

9. Explain how you can defeat a dragon with an odd number of heads and no tails.

10. Explain how to defeat a dragon with 1 head and any number of tails.

    (a) First, assume the number of tails is even. (*Hint:* there are still two different cases to consider here).

    (b) Now suppose that the number of tails is odd. How does your strategy change?

Homework 1: Challenge your family members and friends to defeat dragons created by you!

Homework 2: Math Kangaroo Practice

1. What is the sum of the digits in the number 2009?

   (a) 7
   (b) 11
   (c) 12
   (d) 18

2. Peter has 12 toy cars, and Paul has 4 more toy cars than Peter. How many toy cars do Paul and Peter have together?

   (a) 28
   (b) 16
   (c) 48
   (d) 20

3. After Snow White, the Prince and the 7 Dwarfs ate one apple each, there were 4 apples left in the basket. How many apples were there in the basket before they ate any?

   (a) 8
   (b) 9
   (c) 13
   (d) 14

4. On the last day of school, a father took his three children to the circus. Child tickets were $9 and adult tickets were $12. How much did the father pay altogether for all their tickets?

   (a) $48
   (b) $21
   (c) $39
   (d) $30

5. The doctor gave a certain medicine to Ada. He gave her 60 pills, and told her to take one pill each day. Ada began taking the pills on a Monday. What day of the week will it be when she takes the last pill?

   (a) Monday
   (b) Tuesday
   (c) Wednesday
   (d) Thursday

6. Sophie's mother bought 6 identical boxes of crayons. Sophie took out all the crayons from two of the boxes–there were 18 crayons. How many crayons did her mother buy altogether?

   (a) 26
   (b) 54
   (c) 24
   (d) 108

7. Tom is 2 inches taller than Peter, and 5 inches taller than Paul. How many inches taller is Peter than Paul?

   (a) 7 inches
   (b) 3 inches
   (c) 10 inches
   (d) Paul is taller than Peter

8. In a certain zoo, there were 19 monkeys. Among them were 4 chimpanzees and 3 baboons. The rest of the monkeys were gorillas, which were placed in three cages, the same number of gorillas in each cage. How many gorillas were in one of the cages?

   (a) 5
   (b) 4
   (c) 3
   (d) 6

9. Right now, John is 4 years old, and his father is 26 years old. How old will John's father be when John is 3 times as old as he is now?

   (a) 78
   (b) 38
   (c) 42
   (d) 34

10. Eva bought 2 identical notebooks. She had $4 left over. If she wanted to buy two more notebooks like these, she would need $2 more than she started with. How much did one notebook cost?

   (a) $2
   (b) $10
   (c) $6
   (d) $3

11. Adam, Matt, Paul and Tom were looking at their stamp collections. They found that Matt had more stamps than Paul, and Tom had fewer than Adam. Also, Tom did not have the smallest number of stamps. Which of the boys had the least number of stamps?

    (a) Adam
    (b) Matt
    (c) Paul
    (d) This cannot be determined

12. Father was fathering mushrooms for 2 hours. During the first hour, he found 39 mushrooms. How many mushrooms did he gather during the second hour, if it is known that in 40 minutes mother cleaned all the mushrooms father gathered, and she cleaned 7 mushrooms each 5 minutes?

    (a) 39
    (b) 17
    (c) 74
    (d) 56

13. How many different three-digit numbers can we make that have three different digits using the digits 1, 2, and 3?

    (a) 3
    (b) 6
    (c) 8
    (d) 12