

Homework 3 Problem 2 Solution

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Problem 1

Consider people in the group as vertices. Two people are connected if they are friends with each other. We'll get a graph. Call three people a "friendly trio" if everyone is a friend of another among the three people. It will look like a triangle in the graph. Suppose there are t such friendly trios. Let's count the number of times an edge belongs to a "friendly trio". Each "friendly trio" has three edges in it, so that number is $3t$. Also, we are given that every two people have exactly 7 friends. That means every edge is in exactly 7 "friendly trio"s. Suppose there are e edges, the number of times an edge belong to a "friendly trio" is $7e$. So

$$3t = 7e$$

$$3|7e$$

Since 7 and 3 are primes, by Euclid's Lemma, $3|e$ i.e. the number of total friendships is divisible by 3.