

Problem 4

INDUCTION SHEET: USE ONE PER PROBLEM!

Define the Problem in terms of a statement $P(n)$.

$$P(n): n^2 = 1 + 3 + 5 + \dots + (2n-3) + (2n-1)$$

Also can be written $n^2 = \sum_{i=1}^n (2i-1)$

Prove the statement $P(1)$.

$$P(1): 1^2 = 1 = 1 \quad \checkmark$$

Show that $P(n) \rightarrow P(n+1)$.

Assume $P(n)$, now let's look at $(n+1)^2$

$$(n+1)^2 = n^2 + 2n + 1$$

Since $P(n)$ is true, we can substitute for

$$n^2 = 1 + 3 + \dots + (2n-3) + (2n-1)$$

$$n^2 + 2n + 1 = 1 + 3 + \dots + (2n-3) + (2n-1) + (2n+1)$$

NEXT
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$$= 1 + 3 + \dots + (2(n+1)-5) + [2(n+1)-3] + [2(n+1)-1]$$

$$= (n+1)^2$$

Therefore, $P(n)$ implies $P(n+1)$.

Problem 6

INDUCTION SHEET: USE ONE PER PROBLEM!

Define the Problem in terms of a statement $P(n)$.

$$P(n): 1+2+4+\dots+2^{n-1}+2^n = 2^{n+1}-1$$

Also can be written

$$\sum_{i=0}^n 2^i = 2^{n+1}-1$$

Prove the statement $P(1)$.

$$P(1): 2^0+2^1 = 1+2 = 3 = 2^2-1 = 4-1 = 3 \quad \checkmark$$

$$P(0): 2^0 = 1 = 2^1-1 = 2-1 = 1$$

Show that $P(n) \rightarrow P(n+1)$.

Assume $P(n)$, now let's look at $\sum_{i=0}^{n+1} 2^i = 1+2+\dots+2^n+2^{n+1}$

$$(1+2+\dots+2^n) + 2^{n+1} = 2^{n+1} + 2^{n+1} - 1 = 2 \cdot 2^{n+1} - 1$$

~~REMEMBER~~ REMEMBER:

$$a^m \cdot a^n = a^{(m+n)}$$

$$2^1 \cdot 2^{n+1} = 2^{n+2}$$

From $P(n)$, we can substitute

$$1+2+\dots+2^n = 2^{n+1}-1$$

$$\text{SO: } 1+2+\dots+2^n+2^{n+1} = 2^{n+2}-1$$

Therefore, $P(n)$ implies $P(n+1)$

Problem 8

INDUCTION SHEET: USE ONE PER PROBLEM!

Define the Problem in terms of a statement $P(n)$.

Define: $n! = (n) \cdot (n-1) \cdot (n-2) \dots (3) \cdot (2) \cdot (1)$

$P(n)$: The number of ways to rearrange n distinguishable objects in a row is $n!$

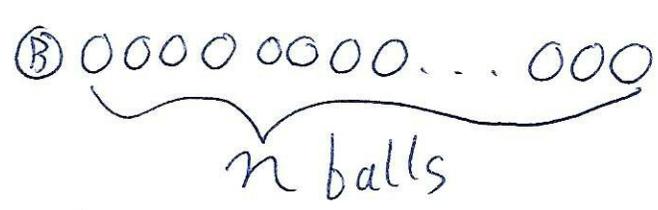
Prove the statement $P(1)$.

$P(1)$: If we have a single object, there is only one way to rearrange it, which is itself, by itself. $1 = 1!$ ✓
on the row

Show that $P(n) \rightarrow P(n+1)$.

Assume $P(n)$, let's look at $n+1$ objects.

~~Imagine~~ Imagine $n+1$ different colored balls, now let's put the blue one in front



Holding the blue ball in front, $P(n)$ says there are $n!$ ways of rearranging the other balls.

Now ~~we~~ let's swap the blue and red balls, we find another $n!$ ~~combinations~~ combinations while holding red ~~in~~ in front. Doing this with all $n+1$ balls gives us all our combinations as

$$\underbrace{n! + n! + \dots + n! + n!}_{n+1 \text{ times}} = (n+1)n! = (n+1)!$$

So $P(n)$ implies $P(n+1)$

Problem 9

INDUCTION SHEET: USE ONE PER PROBLEM!

Define the Problem in terms of a statement $P(n)$.

$P(n)$: The total ways of choosing a group (of any size) ~~is~~ from n students is 2^n .

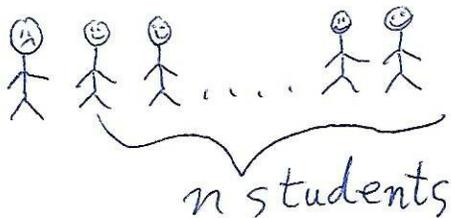
Prove the statement $P(1)$.

$P(1)$: There are $2^1=2$ ways to choose any group from one student. ~~The~~ Either we choose the student or we choose none.

Show that $P(n) \rightarrow P(n+1)$.

Assume $P(n)$, now let's look at $n+1$ students

Imagine $n+1$ students in a line



If we don't include the first student in any groups, then we are choosing from n choices which has 2^n ~~the~~ groups. If we then always choose the first student, then we also only have n choices, so its another 2^n groups.

$2^n + 2^n = 2^{n+1}$
So $P(n)$ implies $P(n+1)$

If you **INSIST** that a group has at least one student, then proceed using $2^n - 1$ instead of 2^n .