

Lesson 2 Problem 5 Solution

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Problem 5.

Let us count how many trio of people can be assembled in total, regardless of relationships. Some of you might remember that this number can be computed as $10 \text{ choose } 3$

$$\binom{10}{3} = \frac{10!}{7!3!} = 120$$

from one of last year's topics. If not, it is easy to get this number by hand: the first person in a triple can be chosen in 10 ways, the second in 9 and the third in 8. Except in a trio we do not care about the ordering of people, so in $10 \cdot 9 \cdot 8$ we overcount every triple 6 times – the number of ways to assign who was chosen first, who second and who third. So the total number of trios is $10 \cdot 9 \cdot 8 / 6 = 120$. Now let us count how many triples have at least one bad relationship in them. Every bad relationship spoils at most 8 triples, as there are 8 people one can add to make a triple with 2 people who don't like each other. With 14 bad relationships, that means at most $8 \cdot 14 = 112$ spoiled triples. But there are 120 in total, so at least 8 are good – and we needed to find at least one.