Take Away Games II: Nim

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The Rules of Nim

- This is a two player game.
- There are piles (rows) of chips.
- During a single turn, a player must select a pile and take at least one chip from that pile. However, a player can take more than one chip from a single pile if they choose.

Game 0: One Pile

To check if you've fully grasped the rules, we'll play two easy games. Make a single pile of as many chips as you would like.

1. Suppose the player who takes the last piece wins. Which player has a winning strategy (P1 or P2)?

2. If the player who takes the last piece *loses*, who will win the game. Which player has a winning strategy (P1 or P2)?

Game 1: Two Equal Piles

You start with two equal piles. The goal is to take the last chip. Play a game with two equal piles and determine a winning strategy.

1. Let each pile have exactly 1 chip. Who will win the game? Which player has a winning strategy? What is the strategy?

2. Is there anything the other player can do in order to win?

3. Does the strategy still work if you start with any sized piles so long as they are equal? Play more games if necessary. If so, what is the strategy for a player to win?

Game 2: Two Unequal Piles

The same basic rules apply to this game as well, but the two piles are not equal in size.

1. Let one pile have two chips and the other have exactly one chip. Which player has a winning strategy? What is their strategy?

2. What can the other player do to try to win?

3. Does the strategy still work if you start with any different sized piles? Play more games if necessary. If so, what is the strategy for a player to win?

And The Winner Is...

1. Say Player A always goes first and Player B always goes second. In Game 1, you started off with two equal piles and figured out who would win. In Game 2, you started off with two unequal piles and figured out who would win. Can the same player win both games?

2. For each position, give the first move. Then decide if this is a winning move or if there is a winning response. A position with n chips in one pile and m chips in the other pile would have position (n,m). For example, if there were two piles, one with 5 chips and the other with 13, then we would denote the starting position as (5,13).

   (a) (2,2)

   (b) (29,27)?

   (c) (n,n)

   (d) (n,n+97)

3. What game state is always a winning state for the player taking their turn?

4. What game state is always a losing state for the player taking their turn?

Game 3: Three Piles

For Game 3, create three piles. Two piles must to have the same number of chips; the third pile can have any number of chips (including having the same number of chips as the other two).

1. If both players play the game perfectly, will the first player or second player win every time?

2. In order to win, what strategy does the winning player use?

3. Who wins if the third pile has the same number of chips as the first two piles?

4. Does it matter how many chips are in the third pile? What about the winning strategy makes this the case?

Game 4: Three Piles, Last Chip Loses

Finally, determine which player will win in the position (1,2,3) if the player who takes the last chip loses.

(Hint: How many moves can the player currently facing this position make?)

Nim Mania!

1. If you started a game with as many piles as you like and each pile had one chip, in what scenario would the first player win? What scenarios would the second player win?

2. In Game 1, you determined who would win if the game started with with two equally-sized piles. Who would win if there were any even number of equally-sized piles? Hint: break the piles into pairs and treat each pair as its own two-pile game.

Last Chip Loses

Determine which player has a win for the following starting positions if taking the last chip is a loss:

1. (n,n)
2. (n,m) $n \neq m$
3. (n,n,k)