

# Lesson 7 Problem 4 Solution

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## Problem 4

b) Assume  $a \geq b$ . Let  $r$  be the remainder for the first iteration of Euclidean algorithm. If  $b \leq a/2$ , then we have  $r \leq a/2$  because of the definition of division. Otherwise,  $b > a/2$ , then we will have  $a = 1 \cdot b + r$  when dividing  $a$  by  $b$  since  $2b > a$ . So

$$r = a - b < a - a/2 = a/2$$

In either case, we have  $r \leq a/2 \leq 50$ . Let  $r'$  be the remainder for the second iteration of Euclidean algorithm, i.e. dividing  $b$  by  $r$ . By the definition of division,  $r' < r \leq a/2 \leq 50$ . We can see that every two iterations of Euclidean Algorithm will make the two numbers at least half. After four steps, the two numbers will be less or equal to 25 and so on. After eight steps, both positive numbers are at most 6. We can write out all the cases: 6 and 5:

$$6 = 5 \cdot 1 + 1$$

$$5 = 1 \cdot 5 + 0$$

6 and 4:

$$6 = 4 \cdot 1 + 2$$

$$4 = 2 \cdot 2 + 0$$

5 and 4:

$$5 = 4 \cdot 1 + 1$$

$$4 = 1 \cdot 4 + 0$$

5 and 3:

$$5 = 3 \cdot 1 + 2$$

$$3 = 1 \cdot 2 + 1$$

$$2 = 1 \cdot 2 + 0$$

5 and 2:

$$5 = 2 \cdot 2 + 1$$

$$2 = 1 \cdot 2 + 0$$

two steps 4 and 3:

$$4 = 3 \cdot 1 + 1$$

$$3 = 1 \cdot 3 + 0$$

two steps 3 and 2:

$$3 = 2 \cdot 1 + 1$$

$$2 = 1 \cdot 2 + 0$$

two steps All the other cases will finish in one step since one is a divisor of the other.

We can conclude that it takes at most  $8 + 3 = 11$  steps.