Fun and Games on a Chess Board

Olga Radko

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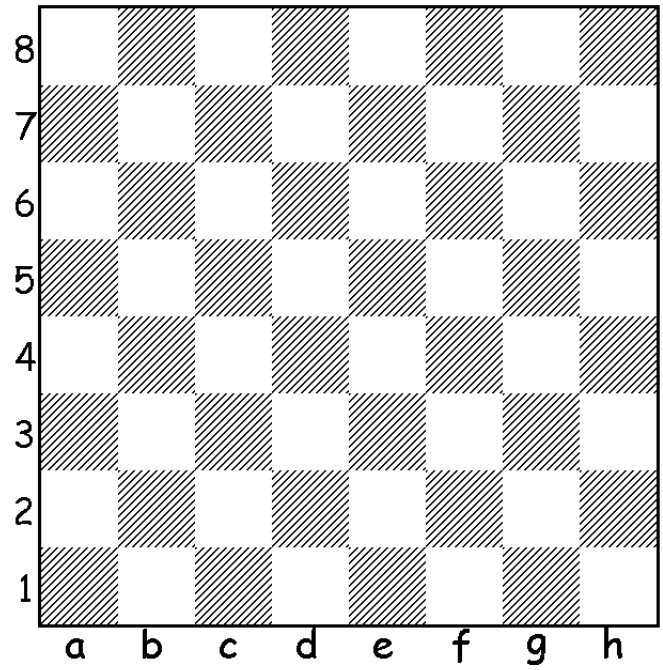
I Names of squares on the chess board

Color the following squares on the chessboard below:

c3, c4, c5, c6, d5, e4, f3, f4, f5, f6



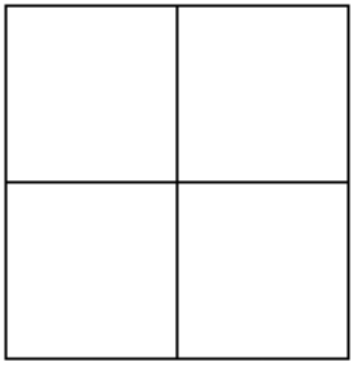
What letter do these squares form together?

II How many squares are there on a chessboard?

A chessboard itself is a square with side 8.

1. The number of 1×1 squares on the chess board is $\square$.

2. What about bigger squares?
   Let's first count squares of size 2×2:

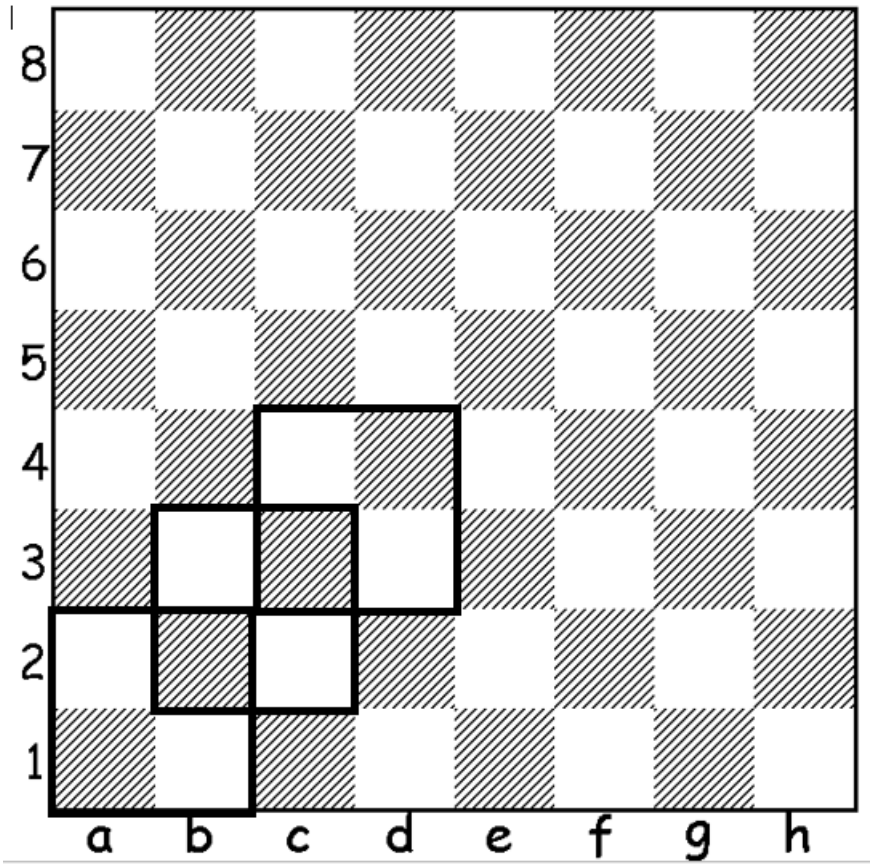


Idea: *Instead of counting* 2×2 *squares, we will count the small* 1×1 *squares which can serve as the left lower corners of the* 2×2 *squares that fit on the chessboard.*

First, shade the left lower corner of the 2×2 square above.

For each of the squares below, decide if it can be a left lower corner of a 2×2 square:

Remember, the 2 x 2 squares on the chessboard can overlap. Like this:



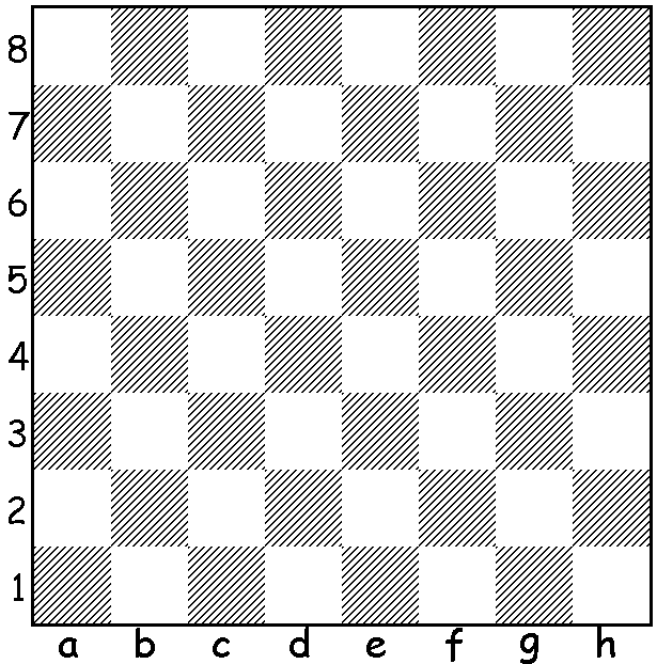
(a) square c3 Yes No

(b) square g6 Yes No

(c) square f8 Yes No

(d) square h2 Yes No

Now color *all* 1×1 squares that can serve as the left lower corners of a 2×2 square:



How many 2×2 squares can you fit onto a chessboard?

3. For each of the squares below, decide if it can be a left lower corner of a 3×3 square:

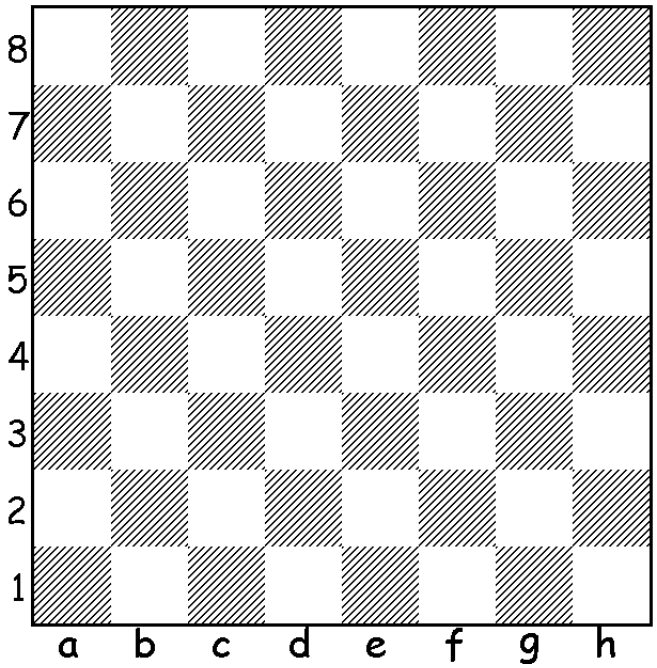
(a) square e6 Yes No

(b) square g3 Yes No

(c) square a7 Yes No

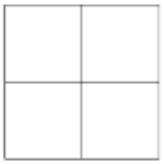
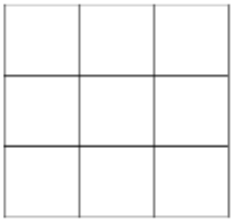
(d) square f6 Yes No

Now color *all* 1×1 squares that can serve as the left lower corners of a 3×3 square:



How many 3×3 squares can you fit onto a chessboard?

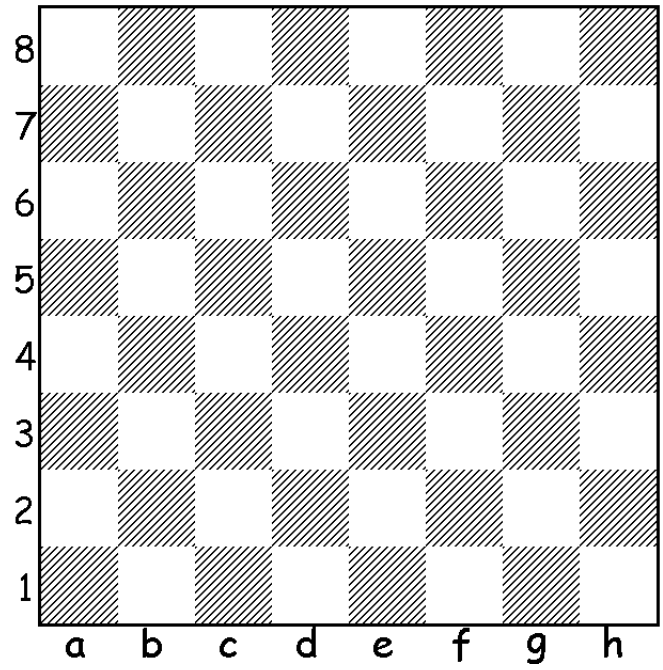
Now you can fill out the table below:

Type of Square	Number of such squares
	
	
	

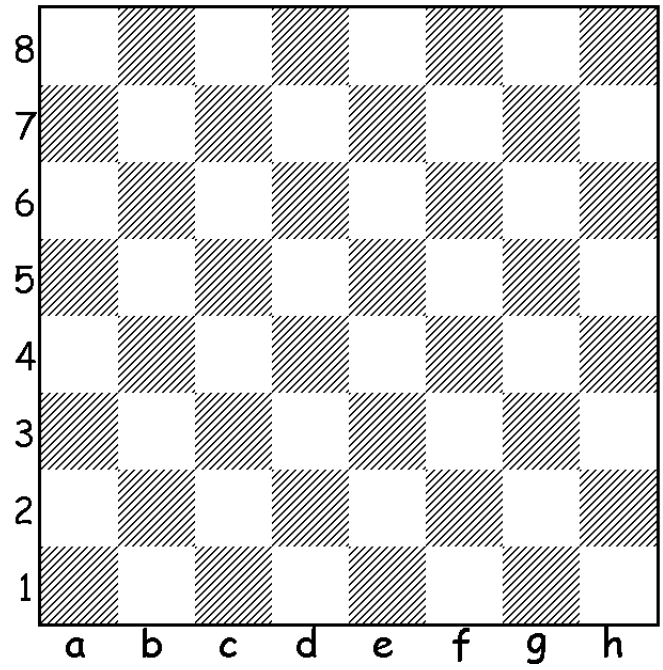
Homework

Count the number of 4×4, 5×5, 6×6 and 7×7 squares on the chessboard in the same way. In each case, use a chessboard picture to shade all the 1×1 squares that can be left lower corners of the bigger squares that fit completely onto the chessboard.

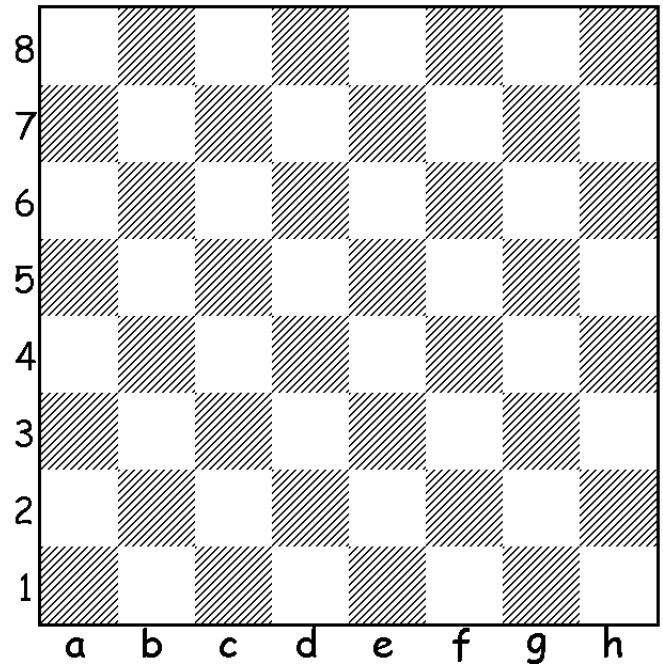
Now color *all* 1×1 squares that can serve as the left lower corners of a 4×4 square:



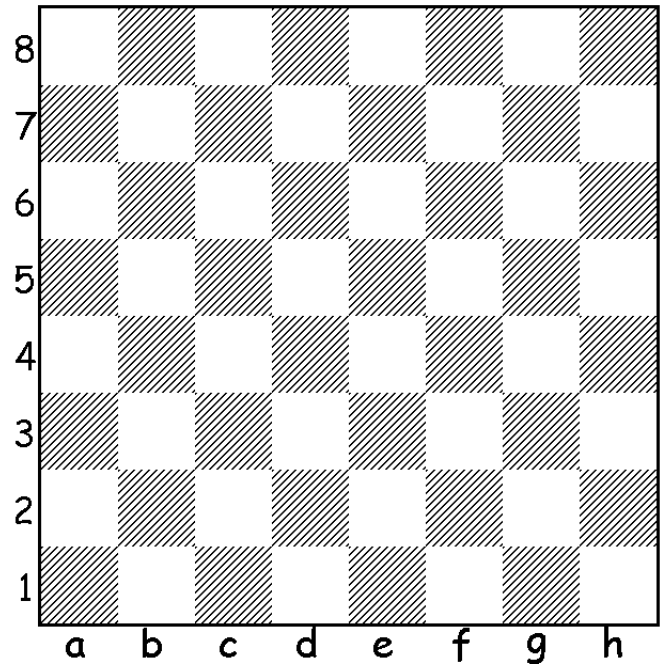
Color all 1×1 squares that can serve as the left lower corners of a 5×5 square:



Color all 1×1 squares that can serve as the left lower corners of a 6×6 square:



Color all 1×1 squares that can serve as the left lower corners of a 7×7 square:



Fill out the table below with the numbers of squares:

size of the square	# of squares of this size
1×1	
2×2	
3×3	
4×4	
5×5	
6×6	
7×7	
8×8	

Now add up all the numbers in the right column to find the total number of squares of all sizes.

II *Rook Race* Game

Two players are playing the following game:

- **Initial position:** Two rooks are placed on two squares of a chessboard.
- **Move:** Move *any* of the rooks to the right by any number of squares.
- **Goal:** To be the *last* person to reach the rightmost square.

1. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. Only one of the players (first or second) has a winning strategy. You need to find it.
   Here are the initial positions:

   (a) Rook I on f3, Rook II on f6
   Which player can win? (Player I or Player II)

   (b) Rook I on d3, Rook II on d6
   Which player can win? (Player I or Player II)

   (c) In general, if both Rooks are the same number of squares away from the right edge, which player can win? How?

2. Now use the following initial positions:

   (a) Rook I on f3, Rook II on d6
   Which player can win? (Player I or Player II)

   (b) Rook I on e3, Rook II on a6
   Which player can win? (Player I or Player II)

   (c) Rook I on a3, Rook II on b6
   Which player can win? (Player I or Player II)

   (d) Rook I on c3, Rook II on g6
   Which player can win? (Player I or Player II)

   (e) In general, if the rooks are different number of squares away from the right edge, which player can win? How?

   (f) If you think you can handle any Rook race game, please challenge one of the instructors to play with you. The instructor will set up an initial position, and you will have a choice of being Player I or Player II. GOOD LUCK!

II *Put Rook Into the Corner* Game

Two players are playing the following game:

- **Initial position:** One Rook is placed somewhere a chessboard.
- **Move:** Move the Rook down or left by any number of squares.
- **Goal:** To put the Rook into the left lower corner.

3. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. In every position, only one of the players (first or second) has a winning strategy. You need to find it.
Here are the initial positions:

   (a) Rook I c3 Rook II on c5
   Which player can win? (Player I or Player II)

   (b) Rook on d4, Rook II on d6
   Which player can win? (Player I or Player II)

   (c) Rook on d4, Rook II on f7
   Which player can win? (Player I or Player II)

   (d) In general, if the Rook is on the diagonal connecting squares a1 and h8, which player can win? How?

(e) How does the game change if the Rook is placed away from the diagonal? Which player can win now?

Homework: Play both the *Rook Race* and *Put Rook into the corner* games at home with your parents, friends, brothers and siblings.