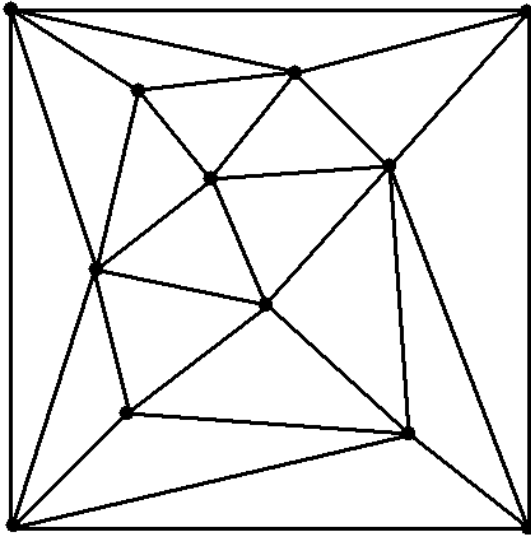


A graph is called *planar* if it can be drawn in such a way that none of its edges cross each other (hence the name “planar”: we can draw it on a flat plane, without edges going over or under each other). Careful with this definition: whether the graph is planar or not does NOT depend on how it’s actually drawn, but only on whether it is POSSIBLE to draw it without edges crossing. Thus the graph below on the left is planar, even though its drawing has crossed edges, since it is actually the same as the graph on the right, which is drawn without crossed edges.



8. There are 7 lakes in Lakeland. They are connected by 10 canals so that one can swim through the canals from any lake to any other. How many islands are there in Lakeland?
9. There are 20 points inside a square. They are connected by non-intersecting segments with each other and with the vertices of the square, in such a way that the square is dissected into triangles. How many triangles do we have?

(This picture is NOT the same as in the problem, it’s just to give you an idea. The problem had 20 points inside the square; below is an example of a square with only 8 points inside.)



10. The asteroid Toroid is shaped like a doughnut. Colonists have built three bio-centers there, and also three utilities (gas, electric, and cable). Each utility needs to be connected with each bio-center by lines across the surface of the asteroid, in such a way that no two lines cross. Can it be done?

