Lesson 5 Problem 5 Solution

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November 12, 2017

Although you could have chosen any odd number other than 3, 9, or 11 which was not divisible by 5, I will only demonstrate the example for $n = 99$ (I'd prefer not to write an infinitely long solution just to account for every single odd integer out there).

Note that $100 = 99 + 1$, so $200 = 2 \cdot 99 + 2$, and in general $m \cdot 100 = m \cdot 99 + m$. Thus, the remainder of $m \cdot 100$ when divided by 99 is the same as the remainder of m when divided by 99. So, if we are given an integer $n = 100 \cdot m + k$ where $0 \leq k < 100$, then n's remainder when divided by 99 is remainder of the sum $100 \cdot m + k$ when divided by 99, which is the same as the remainder of the sum $m + k$ when divided by 99.

Thus, since a number is divisible by 99 if and only if its remainder when divided by 99 is zero, to check if a number is divisible by 99 we only need to check if the sum of the last two digits with the rest of the number is divisible by 99. We can then repeat this process until we get something which is obviously divisible or not divisible by 99. To be more explicit: Suppose we have a number n with an even number of digits: $\overline{a_{2k} \ldots a_1 a_0}$. Then it is divisible by 99 if and only if

$$\overline{a_{2k}a_{2k-1}} + \overline{a_{2k-2}a_{2k-3}} + \ldots + \overline{a_3 a_2} + \overline{a_1 a_0}$$

is divisible by 99. If $n = \overline{a_{2k+1} \ldots a_1 a_0}$ has an odd number of digits, the formula becomes

$$\overline{a_{2k+1}} + \overline{a_{2k}a_{2k-1}} + \overline{a_{2k-2}a_{2k-3}} + \ldots + \overline{a_3 a_2} + \overline{a_1 a_0}$$