

Math Circle Lesson 5 Problem 3

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Across this text, a/b means *remainder of a when divided by b* .

Problem 3.

$n = 11m + 9$ where m is a nonnegative integer. We will look at the remainder of $n/33$ depending on the remainder of $m/3$.

case 1: m is divisible by 3, i.e. $m = 3k$ where k is a nonnegative integer. Then $11m = 33k$ will be divisible by 33, so $n/33$ will have remainder 9.

case 2: m has remainder 1 when divided by 3, i.e. $m = 3k + 1$ where k is a positive integer. Then $n = 11(3k + 1) + 9 = 33k + 20$ will have remainder 20 when divided by 33.

case 3: m has remainder 2 when divided by 3, i.e. $m = 3k + 2$ where k is a nonnegative integer. Then $n = 11(3k + 2) + 9 = 33k + 31$ will have remainder 31 when divided by 33.

We can conclude that 9,20,31 are the only possible remainders. But we also have that n has remainder 2 when divided by 3. Since 33 is divisible by 3, the remainder when divided by 3 will be the same as remainder when divided by 33. In the above three cases, the remainders are 9, 20 and 31. The only one has remainder 2 when divided by 3 is $20 = 3 \times 6 + 2$. So 20 is the only possible remainder of $n/33$