

# Math Circle Lesson 5 Problem 2

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Across this text,  $a/b$  means *remainder of  $a$  when divided by  $b$* .

## Problem 2

a) A positive integer  $n$  that has remainder 7 when divided by 9 will have  $n = 9m + 7$  where  $m$  is a nonnegative integer. Therefore when being divided by 3,  $n = 9m + 7$  will have the same remainder as  $7/3$  since  $9m$  is divisible by 3.  $7/3$  has remainder 1, so clearly  $n$  cannot have remainder 2 when being divided by 3.

b) Suppose it can,  $n = 144m + 23 = 90k + 29$  where  $k$  and  $m$  are nonnegative integers. Consider the remainder of  $n/9$ . When being divided by 9,  $n = 144m + 23$  will have the same remainder as  $23/9$  since  $144m$  is divisible by 9 (This can be checked with the divisibility criterion of 9:  $1 + 4 + 4 = 9$ ).  $23$  has remainder 5 when divided by 9. With similar argument,  $n = 90k + 29$  will have the same remainder as 29 when divided by 9.  $29$  has remainder 2 when divided by 9, which is a contradiction.