Do You Believe in Logic?

1. Clint's mother said, "All champions are good at math." Clint says: "I am good at math. Therefore I am a champion!" Is Clint's reasoning correct?

2. There are four cards on the table. Each has a number on one side and a letter on the other side. The visible sides of the cards show the symbols A, B, 4, and 5. What is the minimum number of cards you must turn over (and which ones) in order to decide whether the following statement is true: "If an even number is written on one side of a card, then a vowel is written on the other side."?

3. Helen bought a rare baseball card for $700, sold it for $800, bought it back for $900, and sold it again for $1000. How much profit did she make?

4. During a trial in Wonderland the March Hare claimed that the cookies were stolen by the Mad Hatter. Then the Mad Hatter and the Dormouse gave testimonies which, for some reason, were not recorded. Later on in the trial it was found out that the cookies were stolen by only one of these three defendants, and moreover, only the guilty one gave true testimony.

   Who stole the cookies?

On the Island of Nod, each inhabitant is either a Knight or a Knave. Knights always tell the truth, and Knaves always lie. As a tourist on the Island of Nod, you are interested to learn which of the friendly natives you meet are Knights, and which are Knaves.

5. You encounter a fellow who proudly tells you, "I am a Knave." Is he a Knight, a Knave, or just another tourist like yourself?

6. Going down the road a bit, you see two more gentlemen who are clearly islanders—let's call them A and B. A tells you, "At least one of us is a Knave." Is A a Knight or a Knave? What about B?

7. You come to a crossroads. You know that one of the roads leads to the City of Knights, and one to the City of Knaves, but someone has stolen the sign which says which road is which. Luckily for you, you spot an islander coming your way. "Excuse me!" you say. She gives you an impatient look—maybe you should be quick. Without knowing whether she is a Knight or a Knave, can you ask a single yes/no question and learn which road is which?

8. You encounter three islanders, who say the following:

    A: "I like strawberries."
    B: "A is a Knight."
    C: "All three of us are Knaves."

    Which of them are Knights, and which are Knaves? Does A like strawberries?

As it happens, some inhabitants of the Island of Nod are werewolves. On nights with a full moon, they turn into wolves and eat people. Werewolves can be either Knights or Knaves. Tonight is a full moon, so you are very anxious to learn which islanders are werewolves and to stay out of their way once the sun goes down.

9. You're interviewing three islanders, and it's known that exactly one of them is a werewolf. The three islanders make the following statements:

   A: "I am a werewolf."

   B: "I am a werewolf."

   C: "At most one of us is a Knight."

   Which of the three are Knights, and which are Knaves? Which one is the werewolf?

10. Now you're interviewing three different islanders, and you know that there is exactly one werewolf among them, and the werewolf is a Knight. You get the following statements:

    A: "At least one of the three of us is a Knave."

    B: "C is a knight."

    Who is the werewolf?

(THERE IS ANOTHER PROBLEM ON THE BACK! —>)

11. Two Math Circle participants run into each other many years later—in fact, they haven't seen each other for over 20 years.

Alice: Bob, is that you!? It's been so long. How are you?

Bob: I'm doing wonderful, thank you for asking! I got married and I have three daughters now.

Alice: Fantastic! What are their ages?

Bob: Well, the product of their ages is 72, and the sum is the same as the number on that building over there.

Alice: (looks at building number, thinks for a minute) Oh, I see... wait, no... Sorry, I still can't figure it out.

Bob: Oops, I didn't give you enough information! My daughter with the highest age is just starting piano lessons.

Alice: Oh, how nice! My son is that same age!

How old are Bob's three daughters?