Lesson 5: More remainders and divisibility.

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Problem 1.

a) Show that a number is divisible by 2 if and only if its last digit is even.

b) Show that a number is divisible by 4 if and only if its last two digits make a number divisible by 4.

c) Can you generalize these principles to make a divisibility criterion for any 2^n?

d) Can you do the same for 5^n?

Problem 2.

a) A positive integer n has remainder 7 when divided by 9. Can it have remainder 2 when divided by 3?

b) A positive integer n has remainder 23 when divided by 144. Can it have remainder 29 when divided by 90?

Problem 3.

A positive integer n has remainder 2 when divided by 3 and remainder 9 when divided by 11. What will be its remainder when divided by 33? Find all possible answers and show that none other exist.

Problem 4.

a) How many zeros does the number 10! end with? Reminder: $n!$ reads n *factorial* and equals $1 \cdot 2 \cdot \ldots \cdot n$

b) Same question for 100!

Problem 5.

Over the last two weeks we have seen the divisibility criterion for powers of 2, as well as for 3, 9 and 11. Find a divisibility criterion for some other odd integer $n > 1$ of your choosing. The only other restriction is that your chosen n should not be divisible by 5.