

Junior Circle Meeting 5 – Probability

May 2, 2010

1. We have a standard coin with one side that we call “heads” (H) and one side that we call “tails” (T).
 - a. Let’s say that we flip this coin 100 times.
 - i. How many times do you expect to get “heads” as an outcome?
 - ii. In an actual experiment, can one get a different number of “heads” when flipping a coin 100 times?
 - iii. Do you think it is likely to get only 10 “heads”?
 - iv. Do you think it is likely to get 55 “heads”?
 - b. Now we flip this coin 1000 times. How many times do you expect to get “heads”?
 - c. How is the number of “heads” you expect to get related to the number of flips?

d. We say that the probability of getting “heads” is $\frac{1}{2}$. A probability is:

$$\frac{\text{number of ways to get something}}{\text{total number of possibilities}}$$

i. What does the “1” in the numerator mean?

ii. What does the “2” in the denominator mean?

iii. What is the probability of “tails” if you flip a coin once?

2. Now let’s flip two coins at the same time.

a. What are all the possible outcomes? (Denote “heads” by H and “tails” by T when listing the outcomes)

b. What is the probability of getting two “heads”?

- d. What is the probability of getting exactly 1 “head”?
- e. What is the probability of getting 0 “heads”?
- f. What is the probability of getting at least two “heads”?
(*At least two* means two or more).
- g. What is the probability of getting at most 1 “head”? (*At most one* means one or less).
- h. What is the relationship between your answers in parts (f.) and (g.)?

4. We roll a single die.

a. What are the possible outcomes of our roll?

b. What is the probability of rolling a 3?

c. What is the probability of rolling an even number?

d. What is the probability of rolling an odd number?

e. What is the relationship between your answers in parts (c.) and (d.)?

- f. What is the probability of rolling at most a 4?
- g. What is the probability of rolling at least a 5?
- h. What is the relationship between your answers in parts (f.) and (g.)?
- i. Based on your answers in parts (e.) and (h.) above, what can you say the sum of probabilities of all outcomes of an experiment?

5. We color the faces of a single die: 4 faces are red and 2 faces are blue.

- a. Olga says that since we colored the die with two colors, it follows that the probability of rolling a red face is $\frac{1}{2}$ and the probability of rolling a blue face is $\frac{1}{2}$. Is she correct?

- b. What are the correct probabilities for rolling a red face and rolling a blue face?

6. Create your own probability problem and solve it here!

7. Now we roll 2 dice, one after another.

a. What is the number of possible outcomes? (You might want to list the outcomes so that it's easier to count).

b. What is the probability of first rolling a 1 and then rolling a 2?

c. What is the probability of first rolling a 2 and then rolling a 1?

- d. What is the probability that both rolls give the same number?
- e. What is the probability that the number on the second roll is strictly larger than the number on the first roll?
- f. What is the probability that the first roll is an odd number?
- g. What is the probability that both numbers are odd?

8. We roll two dice at the same time and find the sum of the two numbers we rolled. What answer will we get most often?

Imagine this as a game: your friend rolls two dice and finds the sum of the numbers. You need to predict what sum your friend will get. If you predict correctly, you win the game! What number should you choose so that you win as often as possible.

- a. What are the possible values for the sum of two numbers that you roll?
- b. Nikki thinks that all the sums are equally likely. Is she correct? Why or why not?

- c. Fill in the table below describing the ways to roll a number (between 2 and 12).

For example, the ways to roll a 3 is filled in for you.

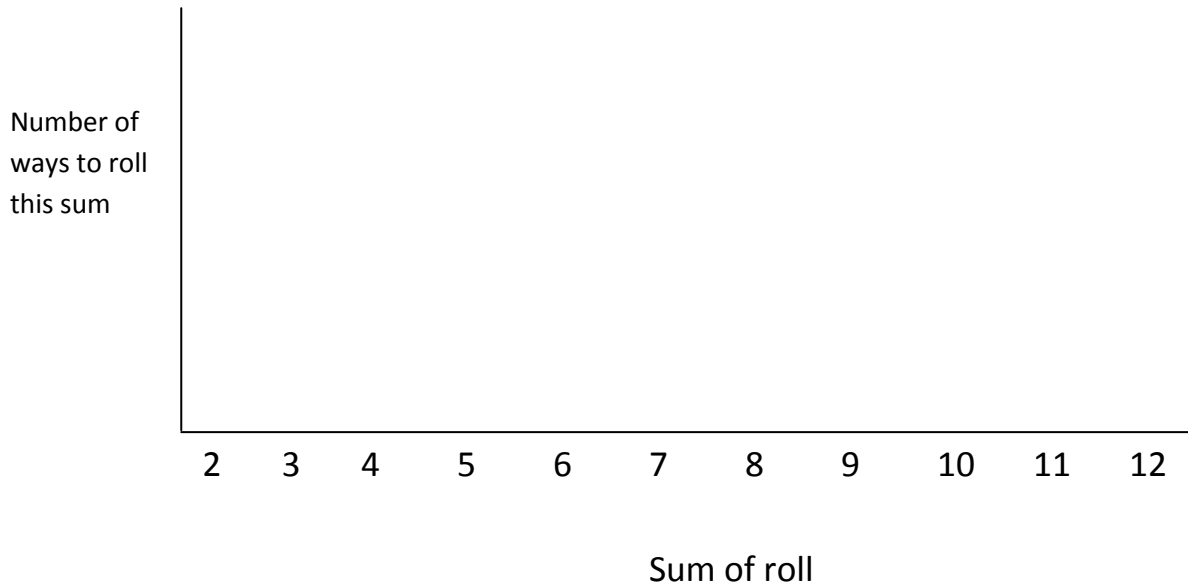
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|----------------|---|---|---|---|---|---|----|----|----|
| | (1,2) (2,1) | | | | | | | | | |

- d. What number or numbers are we most likely to roll? What number or numbers are we least likely to roll?

- e. Compare the probability of getting 3 as the sum to the probability of getting 4 as the sum. Which is more likely?

- f. Make a bar graph below for the number of ways to get a certain number for each of the numbers between 2 and 12.

What do you notice about the shape of your bar graph?



- g. Based on the graph, what number is the most likely one to come up as the sum?

- h. What is the relation between the probability of getting this number and the probability of getting 2?

9. (*Monty Hall Problem*) Suppose you are on a game show, and you're given the choice of three doors: Behind one door is a car and behind the other doors are goats. You pick a door, say door Number 1, and the host, who knows what's behind the doors, opens another door, say door Number 3, which has a goat. He then says to you, "Do you want to pick door Number 2 instead?" Is it to your advantage to switch your choice?

Below is a detailed diagram (from Wikipedia) of the three possibilities of initially picking the car or goat A or goat B. Based on the diagram, do you think the player should switch?

