

## Lesson 4: Algebra and remainders.

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### Problem 1.

- a) The straight line  $y = 7x/15 + 1/3$  passes through two integral points:  $(10, 5)$  and  $(-20, -9)$ . Does it pass through any other integral points?
- b) The graph of a function  $y = kx + b$  passes through two distinct integral points. Are there any other integral points on this graph?
- c) Does there exist a linear function  $y = kx + b$  such that its graph passes through exactly one integral point?

### Problem 2.

Solve the equation:

$$\begin{cases} \frac{x}{x+1} + y^2 = 4 \\ y^2 - \frac{5x}{x+1} = -14 \end{cases}$$

### Problem 3.

- a) Let  $a, b$  be positive integers. Show that there exist unique nonnegative integers  $q, r$  such that  $a = bq + r$  and  $r < q$ .
- b) Let  $a, b$  be integers. Show that there exist unique integers  $q, r$  such that  $a = bq + r$  and  $0 \leq r < |q|$ .

### Problem 4.

Show that  $n^5 + 4n$  is divisible by 5 for any integer  $n$ .

### Problem 5.

Let  $x, y, z$  be integers such that  $x^2 + y^2 = z^2$ . Show that at least one of  $x, y, z$  is divisible by 3.

### Problem 6.

Is it possible to write 1986 as a sum of 6 squares of odd numbers?