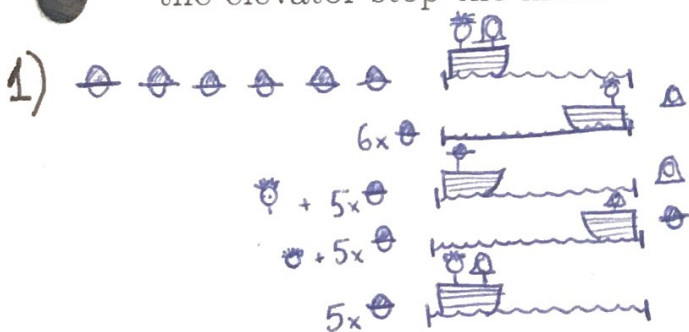


Cutting Logs and Cutting Bagels

October 16, 2017

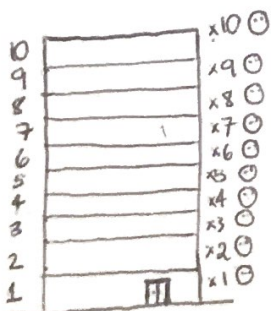
Warm-up problems

1. A squad of six soldiers must cross a deep river with no bridge. They spot two children playing with a small boat by the shore. The boat can only hold two children or one soldier. Can all the soldiers make it across the river?
2. In a ten-story building, only one person lives on the first floor; two people live on the second floor; three people live on the third floor, and so on up to the tenth floor where ten people live. On which floor does the elevator stop the most?



Both children go. One remains on the other side while the other brings it back to the soldiers. The child then stays with soldiers while one of the soldiers takes the boat to the other side. He crosses the river and the first child then brings the boat to the side with the five soldiers and the other child and then repeat the whole process.

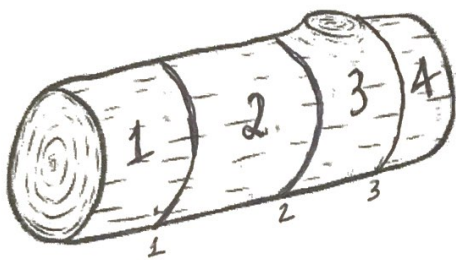
2)



There are 55 people in the building. If each person goes on the elevator one at a time, then the elevator will make 2 stops: 1 on their floor and 1 on the first floor. ~~This means the~~ Assuming everyone uses the elevator once a day, there would be 108 stops (1st floor doesn't use the elevator); 10 on the 10th, 9 on the 9th, 8 on the 8th, ... and 54 on the 1st floor. Therefore the elevator stops on the 1st floor the most.

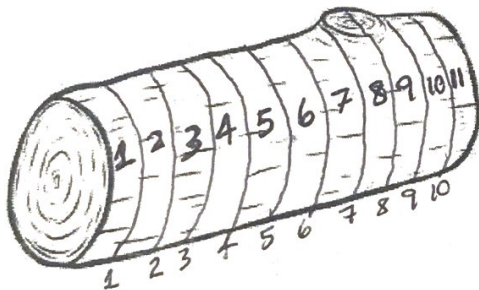
1. Suppose you are cutting a log by using a hand saw.

- How many pieces will you get if you cut a log with 3 cuts?



Answer: You get pieces.

- What about 10 cuts?



Answer: You get pieces.

- What about 27 cuts?

Answer: You get pieces.

- What about 2017 cuts?

Answer: You get pieces.

How is the number of pieces related to the number of cuts?

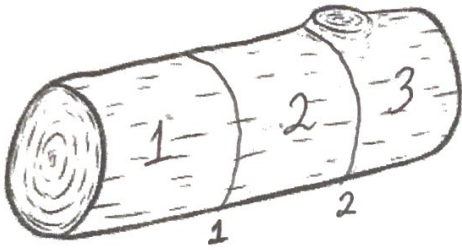
Do you notice a pattern?

$$\text{PIECES} = \text{CUTS} + \boxed{1}$$

Can you explain this?

Every time you cut the log you get however many pieces you had + 1 (because you are splitting 1 piece into 2). Since you start with 1 piece (the log), you will always get $\# \text{pieces} = \# \text{cuts} + \underbrace{\# \text{starting pieces}}_{=1}$

2. How many cuts do you have to make to cut a log into 3 pieces?



2 cuts

- What about 10 pieces?

9 cuts

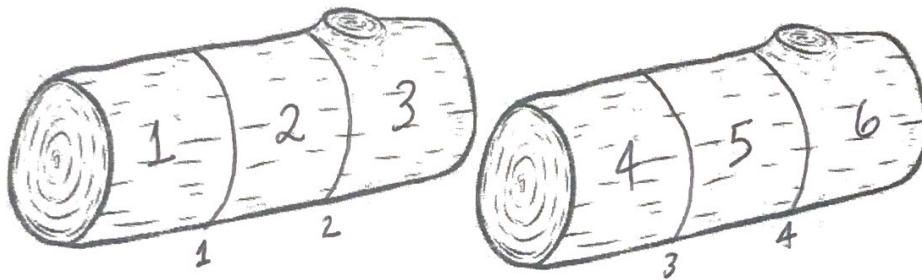
- What is the general rule here?

$$\text{CUTS} = \text{PIECES} - 1$$

3. Now suppose that you have 2 logs. Use 4 cuts to get some number of pieces. It is your choice how many cuts to make on each log, but the total must be 4.

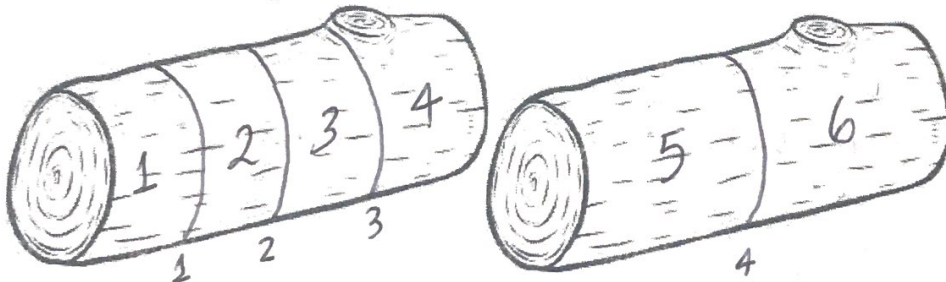
(a) how many pieces do you get?

First attempt:



Answer: you get pieces.

Second attempt:



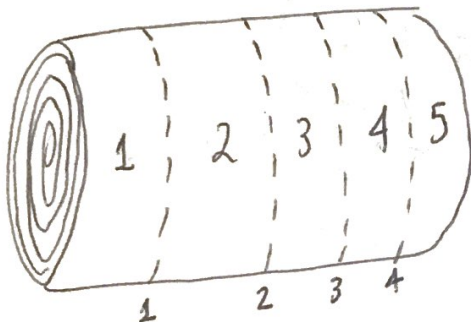
Answer: you get pieces.

- (b) Did you get the same number of pieces each time? Can you explain this?

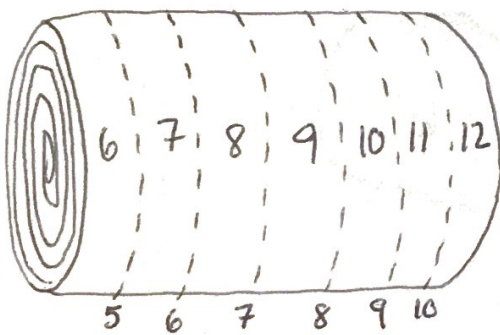
Yes. This follows the same logic as before, except you start with 2 logs, so 2 pieces

4. You still have 2 logs and make some cuts on both of them. The total number of cuts is 10. How many pieces do you get?

Use the space below to make pictures that can help you:



12 pieces



Answer: When you make 10 cuts on 2 logs you get

12

 pieces.

5. Suppose you have 2 logs. How is the number of pieces related to the total number of cuts you make?

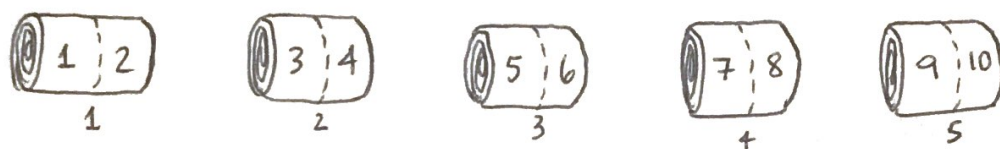
$$\text{PIECES} = \text{CUTS} + \boxed{2}.$$

6. What happens when you cut 3 logs? Investigate this idea, and see what general pattern you can find. $\text{PIECES} = \text{CUTS} + \boxed{3}$.

$$\# \text{ pieces} = \# \text{ cuts} + \# \text{ logs}.$$

7. The Tin Man says that he got 10 pieces after making just five cuts. How many logs did he start with?

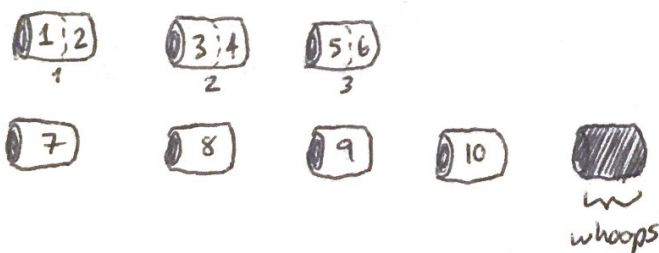
He started with 5 logs



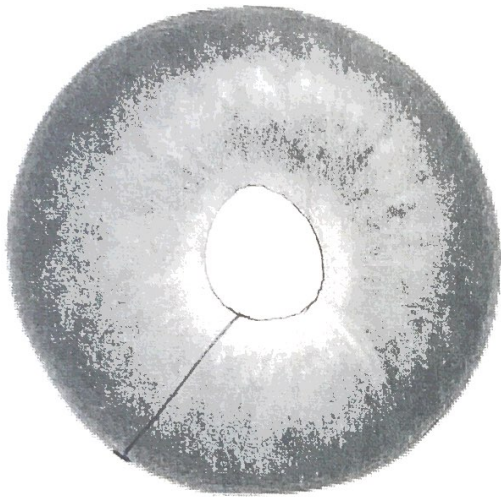
8. The Tin Man says that he can also get 10 pieces of wood by making only three cuts. Do you believe him? If yes, how can he do it? If no, why not? Yes, he can do it by having 7 logs

$$\# \text{ pieces} = \# \text{ cuts} + \# \text{ logs}$$

$$10 = 3 + 7$$



9. Now you have a bagel instead of a log. We will be cutting it in a funny way. A cut goes from the hole to the edge of the bagel. Thus, one cut DOES NOT create two pieces.) Make a picture.



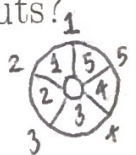
- If you cut a bagel with 3 cuts, how many pieces do you get?

3 pieces



- How many pieces will you get if you cut a bagel with 5 cuts?

5 pieces



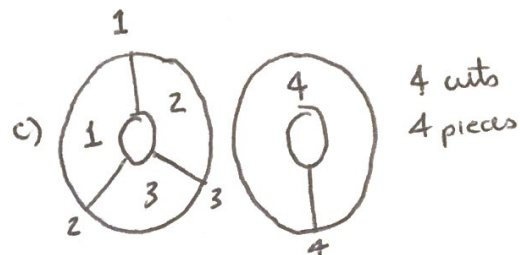
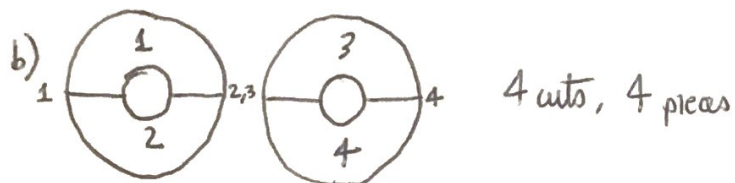
What about 10 cuts?

10 pieces

What happens if you make just one cut? What is the general rule here?
Does it work for any number of cuts?

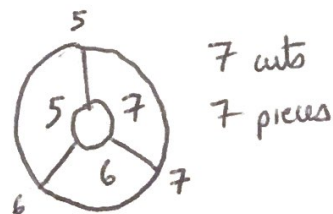
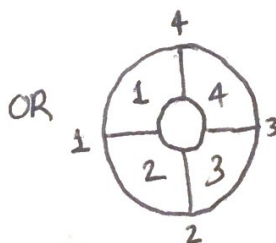
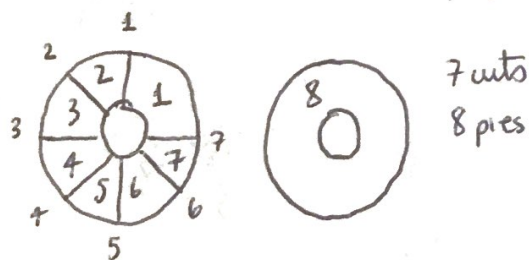
1 cut = 1 piece. For 1 bagel: # cuts = # pieces.

10. Suppose you have 2 bagels. You cut each bagel some number of times so that all together there are 4 cuts. How many pieces do you get? Does it depend on how many cuts you choose to put on each ~~log?~~ bagel



11. What happens if you take two bagels and make a total of 7 cuts on them. How many pieces can you get? Can you come up with a general rule for the number of pieces you can get by cutting two bagels?

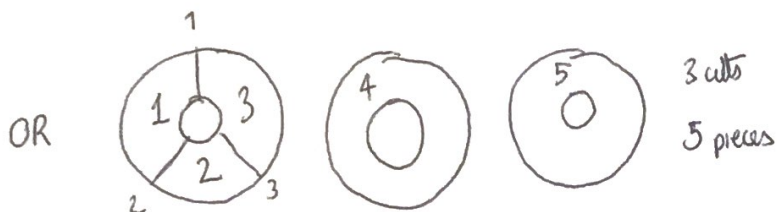
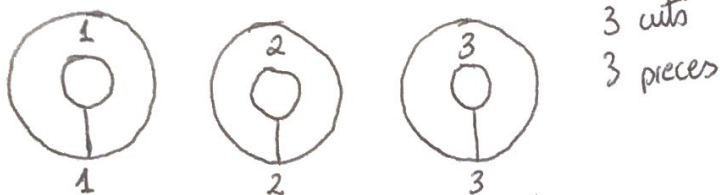
Max number of pieces = # cuts + 1



If you have to have at least 1 cut on each (implying # cuts ≥ 2) bagel, then:

$$\# \text{ pieces} = \# \text{ cuts}$$

12. Now suppose you have 3 bagels and you make 3 cuts. How many pieces can you get?



13. Dorothy says she got 4 bagel slices after making 3 cuts. Find out how many bagels she could have started with.

She could have started with 2 bagels

