# Split the Difference

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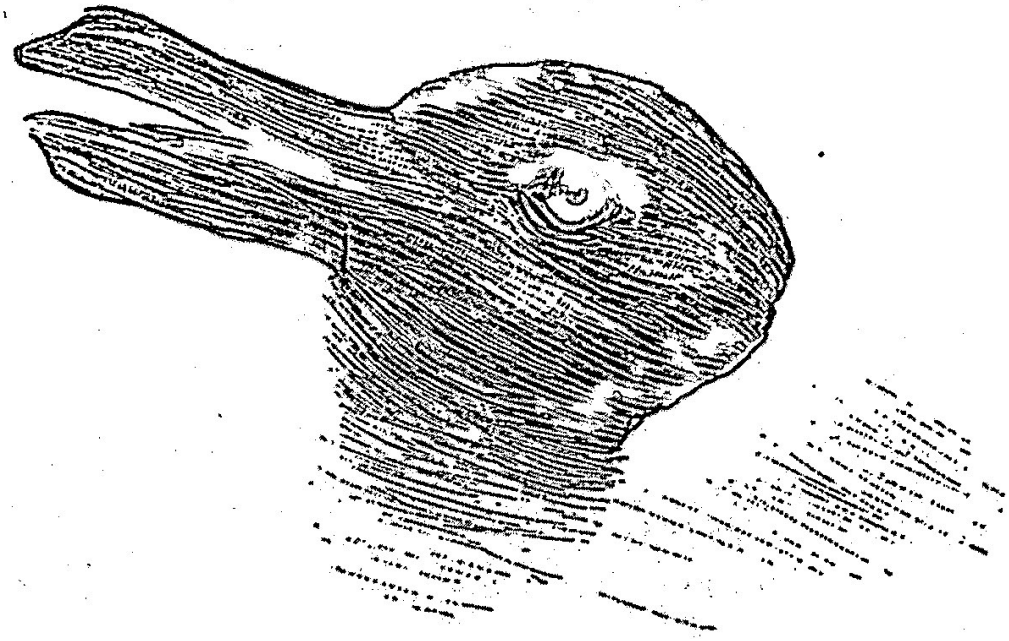
## Warm up: an Optical Illusion

Look at the picture below.



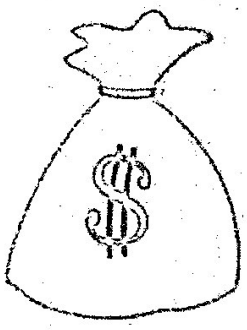
- What does the picture show? A Rabbit and/or a Duck.
- Can you see anything else in this picture? A Duck and/or a Rabbit.
- Does your partner see the same thing you see?
- If not, can both of you be right? Yes, both people can be right!

The picture above is an example of an *optical illusion*. What do you think an optical illusion is? An image that shows multiple things depending on how you look at it.

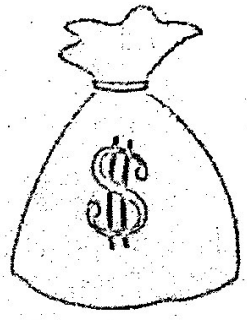
## Sharing Problems

1. Bob and Tom have many coins.
   Bob has 2 more coins than Tom.

Bob:



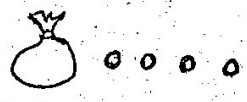
Tom:



How many coins should Bob give to Tom so that they have the same number of coins?

Answer: 1 coin

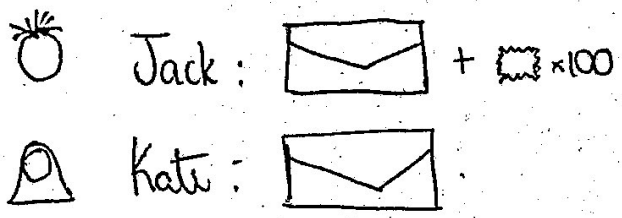
2. Ella has 4 more raisins than Stella.
How many raisins should Ella give to Stella so that they have the same number of raisins?
Make a picture:

Ella: 

Stella: 

Answer: 2 raisins.

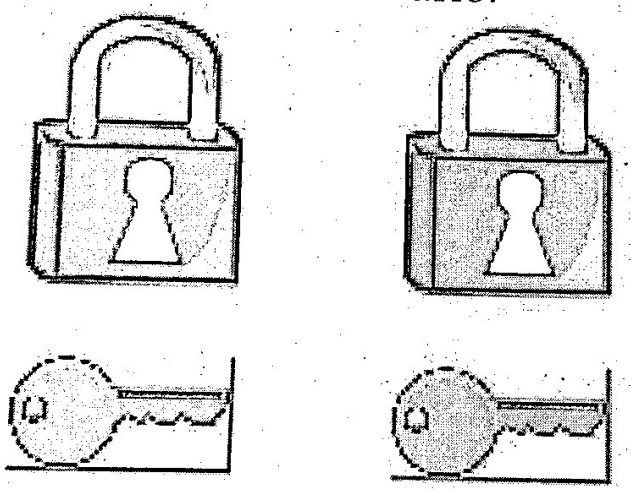
3. Jack has 100 more stamps than Kate.
How many stamps should Jack give to Kate so that they have the same number of stamps?
Make a picture:




Answer: 50 stamps

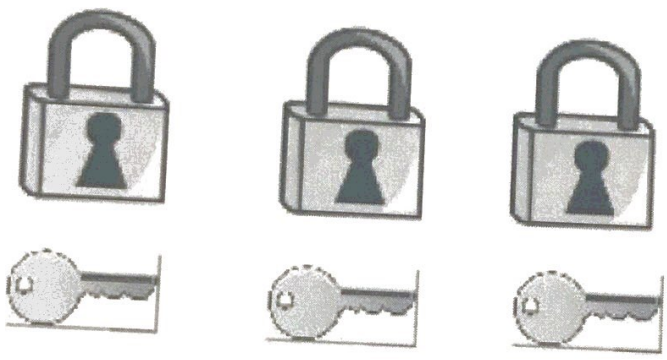
## Locks and Logic

1. There are 2 different keys that open 2 different locks. You want to find out which key opens which lock. On each trial, you insert one key into one of the locks and see if it works or not. How many trials do you need to match the keys with the locks? Consider the worst case scenario.



You only need 1 trial. You choose any 1 of the 2 keys and attempt to open either 1 of the 2 locks. If you get it wrong (1 trial) then you had 1 trial and now know for sure which lock it goes into as there is only 1 other lock left. Then there will only be 1 key and 1 lock left. 1 trial

2. There are 3 different keys that open 3 different locks. How many trials do you need to match the keys with the locks? Think about how this problem is related to the previous one. Consider the worst case scenario.



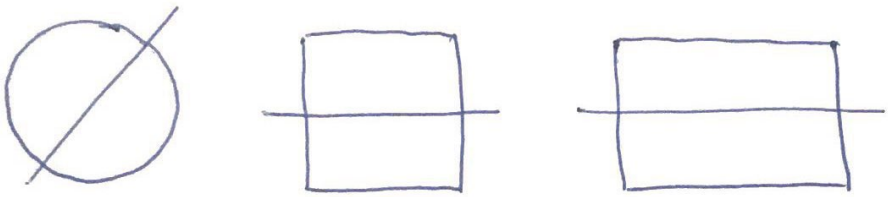
- You choose any 1 of the three keys and try with any 1 of the 3 locks. (1 trial)
- If it's wrong you choose any 1 of the other 2 remaining locks. (2 trials)
- If it's wrong then you for sure know it can only be the lock you haven't tried yet (not a trial)
- Now you choose any 1 of the 2 remaining keys and try it with any 1 of the 2 remaining locks. (3 trials)
- If you get it wrong then you know that your key should go with the other lock and the last remaining key goes with the last remaining lock.

Therefore, at most, in the worst case scenario, you need: **3 trials**

* After you figure out which lock 1 key opens, the problem is reduced to the previous problem.

## Cutting the Birthday cake

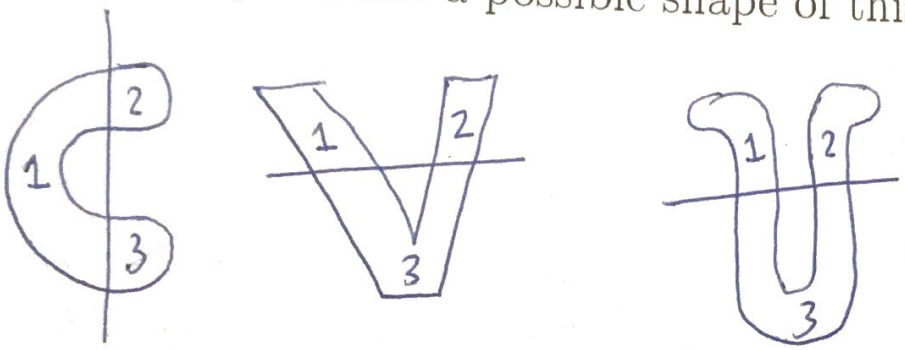
1. What shapes do the birthday cake usually have? Draw several examples. Cut each of the shapes by making one straight cut.



2. How many pieces do you get for each shape after cutting it with one straight cut?

2 pieces

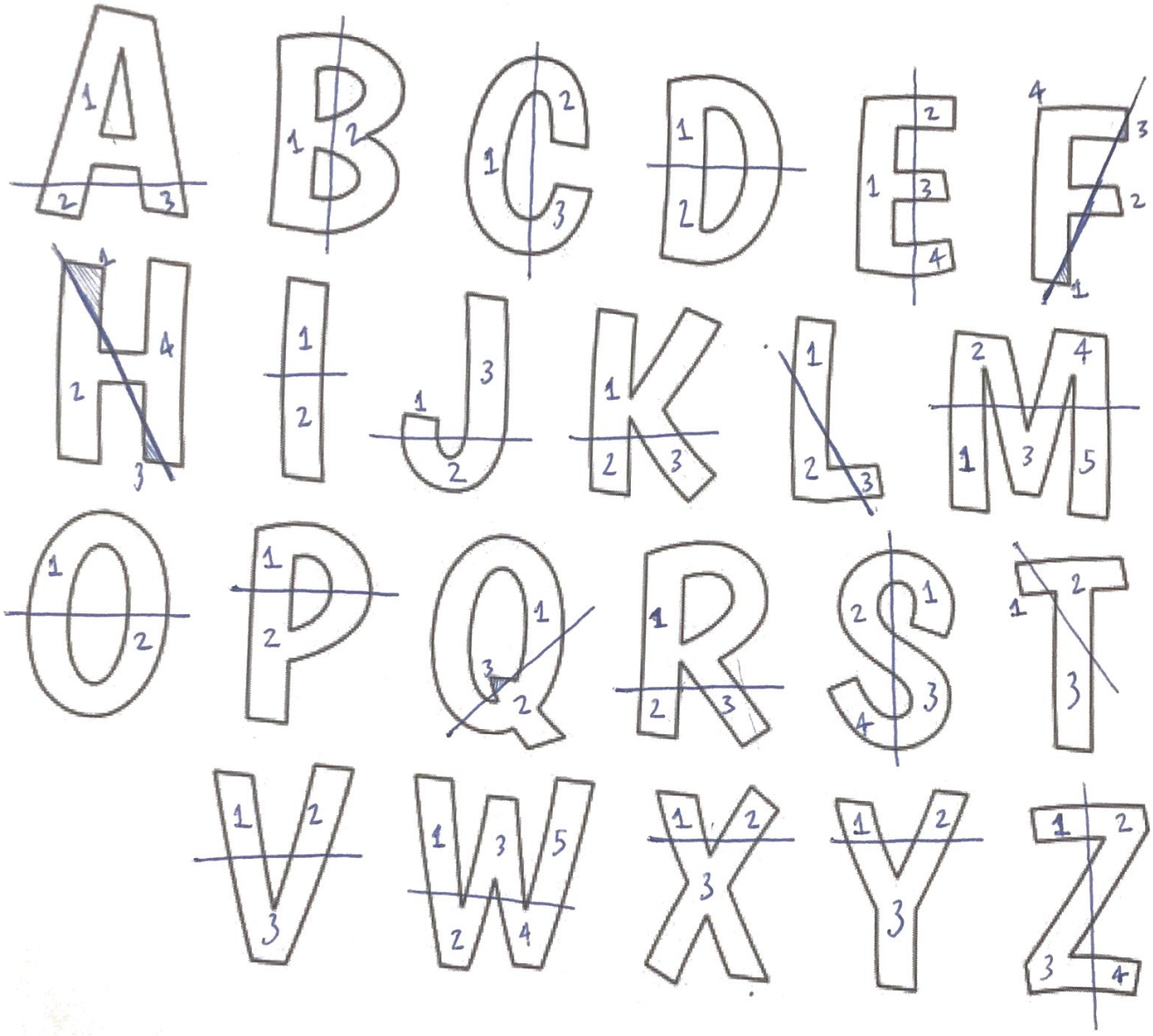
3. Chris's mom baked a cake for Chris's birthday. Chris noticed that the cake has a very interesting shape. You can cut it into 3 parts by making just one cut. Can you draw a possible shape of this cake?



## More to Explore

Imagine that your mom bakes a cake in the shape of the first letter of your name. What is the biggest number of pieces you can cut the cake into by using just one cut? This depends on the letter, of course! Look at all the letters below and cut each of them into the biggest possible number of pieces using just one cut. Several copies of each letter are given for you to practice. You can start with the first letter of your name, or your friend's name, or do it for all the letters. What letter is the winner? That is, for what letter can you cut it into the biggest number of pieces by using exactly one cut? What letters can be cut only into 2 pieces using one cut?

- The letters that can be cut into the largest number of pieces by using only 1 cut are: M and W
- The letters that can only be cut into 2 pieces are:
B, D, I, O, P,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