Egyptian Multiplication

Ancient Egyptians had an interesting method for multiplying two numbers. Suppose that you have to multiply two numbers (e.g., 23 and 18). The basic operation for them was multiplying a number by 2. They reduced all other multiplication problems to it. Here is how they would start multiplying 23 by 18 (in modern notation):

$$23 \cdot 18 : \quad \begin{array}{cccc} 23 & 18 & \otimes & Rows\,used \\ -- & -- & -- & -- \\ \mathbf{1} & \mathbf{18} & = 1 \cdot 18 & \checkmark \\ \mathbf{2} & \mathbf{36} & = 2 \cdot 18 & \checkmark \\ \mathbf{4} & 72 & = 4 \cdot 18 & \checkmark \\ 8 & 144 & = 8 \cdot 18 & \\ \mathbf{16} & \mathbf{288} & = 16 \cdot 18 & \checkmark \end{array} .$$

Here is what they did to complete the multiplication

1. Below the first number (in this case, 23), they would write all of the powers of 2 that are smaller or equal to the number. In the example above, these powers of 2 are 1, 2, 4, 8, and 16.

2. In the second column, they would keep doubling the second number (in this case, 18). This produces the list 18, 36, 72, 144, and 288.

3. After that, they would represent the first number as the sum of the powers of 2 (so that each of the powers of 2 is used at most once).
   For example, if the first number is 23, they would find
   $$23 = 16 + 4 + 2 + 1.$$

   After that, they would mark those rows where these powers of 2 are present in the left column. (In our example, the first, the second, the third, and the fifth rows are marked).

Finally, all there is to do at this point is to add the marked numbers in the second column:

$$\begin{array}{rccc} & 1 & 8 \\ & 3 & 6 \\ & 7 & 2 \\ + \quad 2 & 8 & 8 \\ -- & -- & -- \\ 4 & 1 & 4 \end{array}$$

Thus, the result of the multiplication is 414.

In modern notation, we can rewrite the Egyptian Multiplication algorithm in the following way:

$$23 \cdot 18 = (1 + 2 + 4 + 16) \cdot 18 = 1 \cdot 18 + 2 \cdot 18 + 4 \cdot 18 + 16 \cdot 18$$

1. We will now use Egyptian Multiplication to multiply 13 by 22. Write all powers of two less than 13 in the first column and double each row going down to fill the right-side column.

$$
\begin{array}{cc}
13 & 22 \\
\underline{} & \underline{} \\
 & 22 \\
\\
4 &
\end{array}
$$

(b) Represent 13 as a sum of powers of 2.

(c) Finish the Egyptian Multiplication to find 13 times 22.

(d) When performing your Egyptian Multiplication did you start with the highest or lowest power of 2?

2. Explain how each number in the second column is obtained from the number in the first column.

E.g. How do you get

- 88 from 4 and 22?

- 176 from 8 and 22?

3. Using what you noticed in question 2 do the following:

(a) Rewrite each term in the sum: $18 + 36 + 72 + 144$ as a product of 18 and a power of 2. For example,
$$144 = 8 \cdot 18$$

- $18 =$

- $36 =$

- $72 =$

- $288 =$

(b) Finish the expression on the right side:

- $18 + 36 + 72 + 288 = 18 \cdot 1 + 18 \cdot 2 + +$

(c) What do you notice? Can you simplify this expression by factoring out 18?
$$18 \cdot (1 + 2 + +)$$

4. Multiply the following numbers using Egyptian Multiplication:

(a) 13×41

$$
\begin{array}{cc}
13 & 41 \\
-- & -- \\
1 & 41
\end{array}
$$

(b) 41×13

$$
\begin{array}{cc}
41 & 13 \\
-- & -- \\
1 & 13
\end{array}
$$

1. Given two numbers, which one (smaller or larger) will you use as the first number in Egyptian Multiplication? Why? Give an example to justify your answer.

2. Explain in your own words how Egyptian Multiplication works.

3. With a partner, have a race to see who can multiply numbers faster. One of you must use Egyptian Multiplication and the other must use regular, long multiplication. Race 6 times alternating the type of multiplication you do. Show your work below:

    (a) 25×31

    (b) 38×45

    (c) 12×63

(d) 17×52

(e) 112×85

(f) 256×50