

Voting Systems

High School Circle I

June 4, 2017

Today we are going to start our study of voting systems. Put loosely, a voting system takes the preferences of many people, and converted them into a group preference. The first thing that we are going to do is watch the YouTube video Voting Systems and the Condorcet Paradox — Infinite Series from the channel PBS Infinite Series. The URL is given below:

<https://www.youtube.com/watch?v=HoAnYQZrNrQ>

1. Ok, so that video went through quite a few things rather quickly. Let's take a closer look at what they are saying, and go through things a little more carefully. First, let's define what a voting system actually is.

A voting system is a process whereby preferences of individuals are converted into preferences of a group.

- (a) For the rest of this handout, we are going to be talking about ranked voting. Ranked voting is a kind of voting system where voters take all possible candidates and rank the candidates in order of preference.

What are some other voting systems that are not ranked voting? When are these used? Try and think of at least 2 other voting systems.

- (b) One of the reasons that we spend so much time talking ranked voting systems, is because many other 'different' voting systems can be thought about as a simplification of ranked voting. For example, if your voting system is to just pick your favorite candidate then this can be done by using a ranked voting system and just focusing on the most preferred candidate for each ballot.

For the voting systems that you thought of above, can you look at them as special cases of ranked voting? If so, describe how one could translate a ranked ballot into your voting system. If not, prove that you can't do this translation.

- (c) Here is an example of a voting system, albeit not a terribly good one. No matter what the individual's preferences are, the groups preference is always alphabetical. This is clearly not a terribly good voting system. It isn't fair! One way to mathematize the idea of fairness is to add a fairness criterion.

Let's say that a voting system is 'Majority fair' if the following property holds. If one candidate is the most preferred candidate by more than half (i.e. the majority) of voters, then that candidate should be the group's most preferred candidate.

Is the alphabetical system majority fair? What about the two voting systems that you proposed before, are they majority fair?

- (d) Do you think that it is a necessary for a 'fair' (morally fair, not mathematically fair) system to satisfy the Majority fairness criterion?

2. Now let's return to the voting systems that are described in the video. The video talks about 5 voting systems:

- Plurality - The candidate with the most first place votes wins.
- Run-off - Ignore all candidates except for the two most popular ones (as measured by plurality). Who would win an election if only those two candidates ran?
- Sequential Run-off - Ignore the candidates with the fewest number of first place votes. Remove them from the running and adjust everyone's ballot accordingly. Repeat this process until only one candidate remains. That candidate is the winner.
- Borda Count - For every ballot, give 0 points to that voter's last choice, 1 to their second to last choice, etc... Do this process for every ballot. The candidate with the most votes wins.
- Condorcet - For every possible pair of different candidates, see which candidate would win in a head to head competition. The winner is the candidate which wins the most head-to-head races.

In order to make sure that we really understand what each of these systems are, let's get some practice using them.

- (a) Which location would be the classes' favorite vacation destination, if we used the plurality voting system?

(b) Same question, except using the Run-off method.

(c) Same question, except using the Sequential Run-off method.

(d) Same question, except using the Borda Count method.

(e) Same question, except using the Condorcet method.

(f) Which of the discussed voting systems (if any) obey the majority fairness criterion?

3. Now let's look at some 'real life' applications of these voting schemes.

(a) Suppose that there is a sixth grade class, which has 15 boys and 10 girls. Three students are running for class representative, two boys and one girl. Because this is sixth grade, you know that all of the boys will vote for a boy before any woman, and vice versa. Suppose that the women get to choose the voting system. What system should they choose maximize their chances of a candidate winning?

(b) Same question, but this time suppose that the boys are in charge of deciding the voting system.

(c) Suppose that the High School I LAMC group wants to order a pizza. The possible toppings are sausage (S), pineapple (P) and extra cheese (E). To keep things simple, we are using a plurality voting system. You asked people what they were going to vote for beforehand, and it looks like the votes will come out as:

Number of votes	5	3	7	2
First choice	S	S	E	P
Second choice	E	P	P	E
Third choice	P	E	S	S

If you and your close friend are the voters in the last column, what should you do?

- (d) We say that someone is voting honestly when their ballot really reflects their preferences. Conversely, we say that someone is voting strategically when their ballot does not actually reflect their voting preferences.

When would someone vote strategically instead of honestly? Would it ever be in their advantage. *Hint, it's called strategic for a reason.

- (e) What do you think about strategic voting? Is it moral? Immoral? Write a few sentences.

- (f) Look at the five voting methods that we discussed prior. Are any of those methods immune to strategic voting?

(g) Think back to the voting system that you thought of. Is that immune to strategic voting?

(h) Can you think of a voting system that is immune to strategic voting?
*Hint there is one (it's even deterministic) but it is generally not thought of as a very good voting system for another reason.

4. Alright, let's look at even more real world examples.

(a) The American system for electing officials (e.g. presidential candidates) uses two rounds of voting, not just one. In the first round (called the primaries) the various parties (Democrat, Green, Republican, etc...) select the candidate that they will endorse for president (i.e. who will represent their party in the general election).

In the primaries the winner is chosen by plurality. It's not unusual for the primaries to have a LOT of candidates (in 2016, the Republican party had 12 candidates!). Suppose that you wanted to win the primary election (and maybe later the general election). You have two strategies. You could try to appeal to a lot of voters, but not appeal to them very much (call such a candidate a moderate candidate). Alternatively, you could try and appeal a whole lot to small subset of your party (call this an immoderate candidate).

If you want to win your primaries, what strategy is best?

(b) Suppose that you made it through the primaries, and now you are now your parties candidate in the general election. Suppose that you are running against only 1 other person. Should you try and be moderate, or immoderate?

(c) Often in presidential cycles, candidates (of all parties) start off as very immoderate, and become more moderate as the election cycle goes on. Is this strategy in agreement with your analysis?

- (d) Suppose that the primaries and general election winners were not chosen by plurality, but instead by some other method. How would your answers to the above three questions change?