

Some problems

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Problem 1.

There are 40 cups in a line. Not all of them have the same shape, and not all of them have the same color. Prove that there must be two cups that differ in both shape and color.

Problem 2.

On the island live knights who always tell the truth and liars who always lie. A tourist met three islanders and asked each of them: how many knights are among your two friends? The first one said "None", the second one said "One". What did the third one say?

Problem 3.

Find all even positive integers n with the following property: if n is divisible by a prime p , then $n - 1$ is divisible by $p - 1$.

Problem 4.

Some (at least one) squares of a regular 8×8 chessboard have pawns on them. Prove that there must exist a row and a column such that the total number of pawns in this row and column is odd.