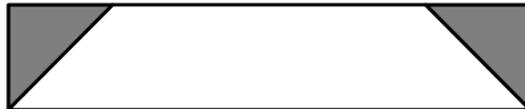


1. Ed and Ann both have lemonade with their lunch. Ed orders the regular size. Ann gets the large lemonade, which is 50% more than the regular. After both consume  $\frac{3}{4}$  of their drinks, Ann gives Ed a third of what she has left, and 2 additional ounces. When they finish their lemonades they realize that they both drank the same amount. How many ounces of lemonade did they drink together?
2. For how many positive integers  $n$  is  $\frac{n}{30-n}$  also a positive integer?
3. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?
4. Kiana has two older twin brothers. The product of their three ages is 128. What is the sum of their three ages?
5. By inserting parentheses, it is possible to give the expression

$$2 \times 3 + 4 \times 5$$

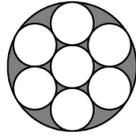
several values. How many different values can be obtained?

6. A rectangular yard contains two flower beds in the shape of congruent isosceles right triangles. The remainder of the yard has a trapezoidal shape, as shown. The parallel sides of the trapezoid have lengths 15 and 25 meters. What fraction of the yard is occupied by the flower beds?

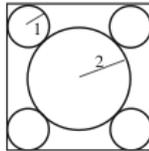


7. In a certain year the price of gasoline rose by 20% during January, fell by 20% during February, rose by 25% during March, and fell by  $x\%$  during April. The price of gasoline at the end of April was the same as it had been at the beginning of January. To the nearest integer, what is  $x$ ?
8. A  $45^\circ$  arc of circle A is equal in length to a  $30^\circ$  arc of circle B. What is the ratio of circle A's area and circle B's area?
9. Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

10. Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



11. A triangle with side lengths in the ratio  $3 : 4 : 5$  is inscribed in a circle with radius 3. What is the area of the triangle?
12. Integers  $a, b, c,$  and  $d,$  not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that  $ad - bc$  is even?
13. Four circles of radius 1 are each tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?



14. Suppose that  $m$  and  $n$  are positive integers such that  $75m = n^3$ . What is the minimum possible value of  $m + n$ ?
15. In trapezoid  $ABCD$  we have  $\overline{AB}$  parallel to  $\overline{DC}$ ,  $E$  as the midpoint of  $\overline{BC}$ , and  $F$  as the midpoint of  $\overline{DA}$ . The area of  $ABEF$  is twice the area of  $FECD$ . What is  $AB/DC$ ?
16. For how many positive integers  $n$  less than or equal to 24 is  $n!$  evenly divisible by  $1 + 2 + \dots + n$ ?
17. Forty slips are placed into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let  $p$  be the probability that all four slips bear the same number. Let  $q$  be the probability that two of the slips bear a number  $a$  and the other two bear a number  $b \neq a$ . What is the value of  $q/p$ ?
18. Triangle  $ABC$ , with sides of length 5, 6, and 7, has one vertex on the positive  $x$ -axis, one on the positive  $y$ -axis, and one on the positive  $z$ -axis. Let  $O$  be the origin. What is the volume of tetrahedron  $OABC$ ?

19. Let  $k = 2008^2 + 2^{2008}$ . What is the units digit of  $k^2 + 2^k$ ?
20. A permutation  $(a_1, a_2, a_3, a_4, a_5)$  of  $(1, 2, 3, 4, 5)$  is heavy-tailed if  $a_1 + a_2 < a_4 + a_5$ . What is the number of heavy-tailed permutations?
21. In the expansion of  $(1 + x + x^2 + \cdots + x^{27})(1 + x + x^2 + \cdots + x^{14})^2$ , what is the coefficient of  $x^{28}$ ?
22. What is the area of the region defined by the inequality  $|3x - 18| + |2y + 7| \leq 3$ ?
23. Points  $A$  and  $B$  lie on a circle centered at  $O$ , and  $\angle AOB = 60^\circ$ . A second circle is internally tangent to the first and tangent to both  $\overline{OA}$  and  $\overline{OB}$ . What is the ratio of the area of the smaller circle to that of the larger circle?
24. Let  $a_1, a_2, \dots$  be a sequence determined by the rule  $a_n = a_{n-1}/2$  if  $a_{n-1}$  is even and  $a_n = 3a_{n-1} + 1$  if  $a_{n-1}$  is odd. For how many positive integers  $a_1 \leq 2008$  is it true that  $a_1$  is less than each of  $a_2, a_3$ , and  $a_4$ ?
25. Rhombus  $ABCD$  has side length 2 and  $\angle B = 120$ . Region  $R$  consists of all points inside the rhombus that are closer to vertex  $B$  than any of the other three vertices. What is the area of  $R$ ?
26. Jim starts with a positive integer  $n$  and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with  $n = 55$ , then his sequence contains 5 numbers:

$$\begin{array}{rcl}
 & & 55 \\
 55 & - & 7^2 = 6 \\
 6 & - & 2^2 = 2 \\
 2 & - & 1^2 = 1 \\
 1 & - & 1^2 = 0
 \end{array}$$

Let  $N$  be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of  $N$ ?

27. Each of 2010 boxes in a line contains a single red marble, and for  $1 \leq k \leq 2010$ , the box in the  $k$ th position also contains  $k$  white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let  $P(n)$  be the probability that Isabella stops after drawing exactly  $n$  marbles. What is the smallest value of  $n$  for which  $P(n) < \frac{1}{2010}$ ?