

Logic Gates!

Early Elementary Math Circle

Computers are made out of devices called “logic gates”. In this handout we’ll learn what logic gates are and what they do.

1 And gates

1. If you combine two true statements using the word “and”, is the new statement true or false?

For example, the statements “grass is green” and “the sky is blue” are both true. Is the following statement true or false?

$\underbrace{\text{Grass is green}}_{\text{Statement 1}} \quad \text{and} \quad \underbrace{\text{the sky is blue.}}_{\text{Statement 2}}$

True

4. If you combine two false statements using the word “and”, is the new statement true or false?

For example:

Grass is blue
 Statement 1

and

the sky is green.
 Statement 2

False

5. Fill in the following table summarizing your answers above.

Statement 1	Statement 2	Statement 1 and Statement 2
False	False	? False
False	True	? False
True	False	? False
True	True	? True

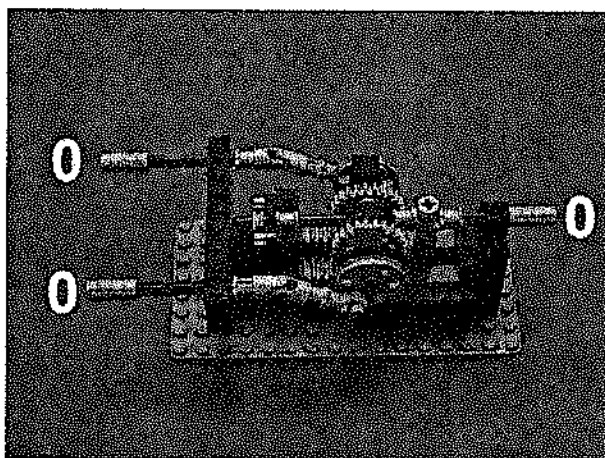


Figure 1: An “And gate” made out of Legos.

An “And gate” is a device that takes two inputs and gives you one output. Each input has two possible values: “True” and “False”. The output has the value “True” if the first input is True *and* the second input is True. Otherwise, the output has the value “False”.

Figure 1 shows an “And gate” made out of Legos. Notice the two inputs on the left. If an input rod is pushed inwards, then that input has a value of “True”. If an input rod is not pushed inwards, then that input has a value of “False”. In Figure 1, both inputs have a value of “False”. (In this picture, the number 0 is used as a short way of writing “False”.)

Normally “And gates” are made out of tiny devices called transistors. A real “And gate” in a computer is so small that it can’t be seen without a powerful microscope. However, you can make them out of Legos also.

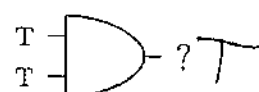
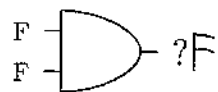
An "And gate" is drawn on paper like this:



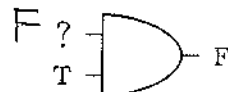
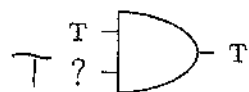
In this picture, the first input has the value "True" ("T" for short) and the second input has the value "False" ("F" for short).

6. What is the output of the And gate in the picture above? Write the correct output on the picture. F

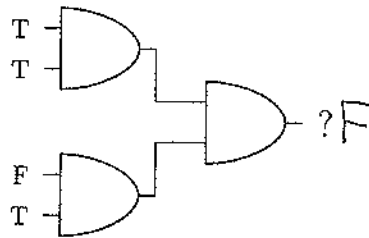
7. What are the outputs of the following And gates?



8. Fill in the missing inputs on the following And gates:



Multiple And gates can be combined by feeding the output of one And gate into the input of another And gate. For example:

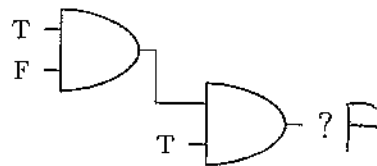


This combination of And gates is an example of a “logic circuit”.

9. What is the output of the above logic circuit?

F

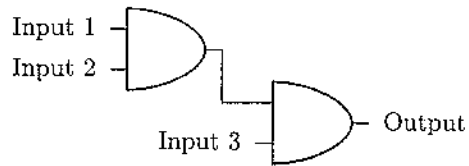
10. What is the output of the following logic circuit?



What values should the three inputs have in order for the output to have the value “True”?

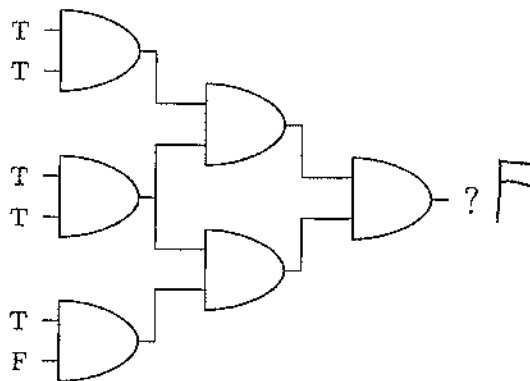
T, T, T

11. Fill in the table below with the correct output values for this logic circuit:

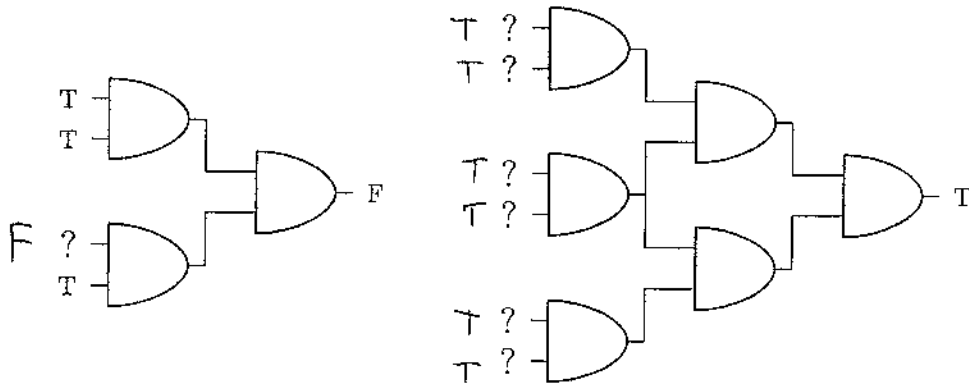


Input 1	Input 2	Input 3	Output
F	F	F	? F
F	F	T	? F
F	T	F	? F
F	T	T	? F
T	F	F	? F
T	F	T	? F
T	T	F	? F
T	T	T	? T

12. What is the output of the following logic circuit?



13. Fill in the missing inputs in the following logic circuits:



2 Or gates

1. If you combine a true statement and a false statement using the word “or”, is the new statement true or false?

For example, the statement “the sun is a star” is true, and the statement “the moon is a star” is false. Is the following statement true or false?

$\underbrace{\text{The sun is a star}}_{\text{Statement 1}} \quad \text{or} \quad \underbrace{\text{the moon is a star}}_{\text{Statement 2}}$

True

4. If you combine two true statements using the word “or”, is the new statement true or false?

For example:

$\underbrace{\text{The sun is a star}}_{\text{Statement 1}} \quad \text{or} \quad \underbrace{\text{the earth is a planet.}}_{\text{Statement 2}}$

True

Warning! The way mathematicians use the word “or”, the above statement is considered to be **true**.

In other words, when mathematicians make a statement like “A is true or B is true”, they mean that either A is true, or B is true, *or possibly both* A and B are true.

This is not always the way people use the word “or” in everyday speech.

5. Fill in the following table summarizing your answers above.

Statement 1	Statement 2	Statement 1 or Statement 2
False	False	? False
False	True	? True
True	False	? True
True	True	? True

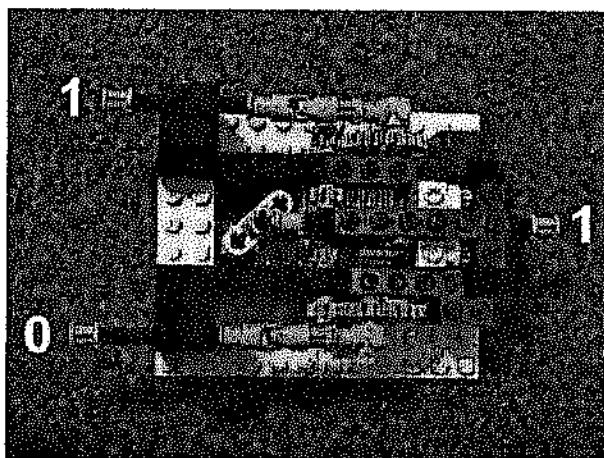
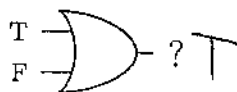


Figure 2: An “Or gate” made out of Legos. In computers Or gates are made out of “transistors” and are extremely tiny, about 1/1000 the width of a human hair. Notice that one input rod on the left is pushed inward, signifying “True”, and the other input rod is not pushed inward, signifying “False”.

An “Or gate” is a device that takes two inputs and gives you one output. Each input has two possible values: “True” and “False”. The output has the value “True” if either the first input is True *or* the second input is “True”. (If both inputs are True, then the output is True – remember the warning above.) Otherwise, the output has the value “False”.

An “Or gate” is drawn on paper like this:



In this picture, the first input has the value "True" ("T" for short) and the second input has the value "False" ("F" for short).

6. What is the output of the Or gate in the picture above? Write the correct output on the picture.

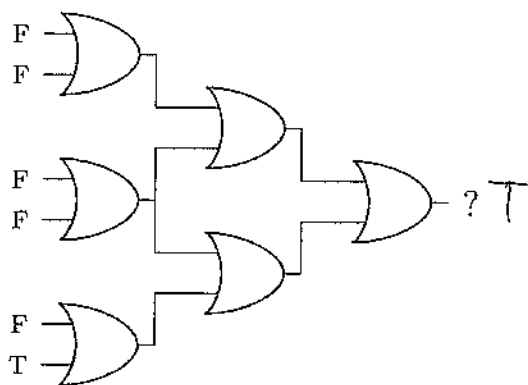
7. What are the outputs of the following Or gates?



8. Fill in the missing inputs on the following Or gates:



9. What is the output of the following logic circuit?



3 Not gates

1. If you modify a true statement using the word “not”, is the new statement true or false? For example, the statement “ice is solid” is true. Is the following statement true or false?

Ice is **not** solid. *False*

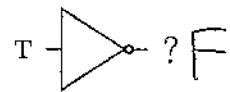
2. If you modify a false statement using the word “not”, is the new statement true or false? For example, the statement “ice is liquid” is false. Is the following statement true or false?

Ice is **not** liquid. *True*

3. Fill in the blanks: When you modify a statement using the word “not”, then true statements become false and false statements become true.

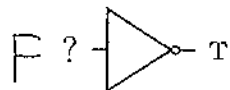
A “Not gate” is a device that takes *one* input and gives you one output. The input has two possible values: “True” and “False”. The output has the value “True” if the input is “False”, and the output has the value “False” if the input is “True”.

A “Not gate” is drawn on paper like this:



In this picture, the input has the value “True” (“T” for short)

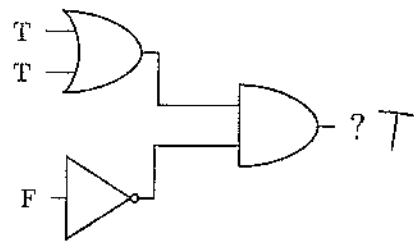
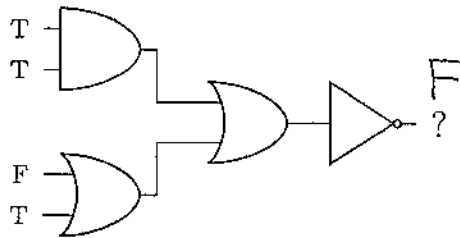
4. What is the output of the Not gate in the picture above? Write the correct output on the picture. F
5. Fill in the missing input in the following Not gate:



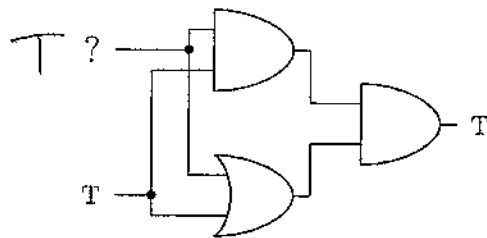
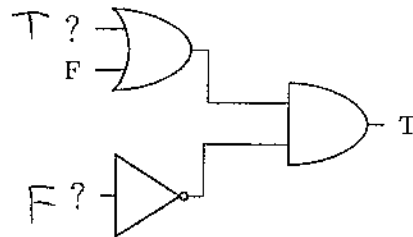
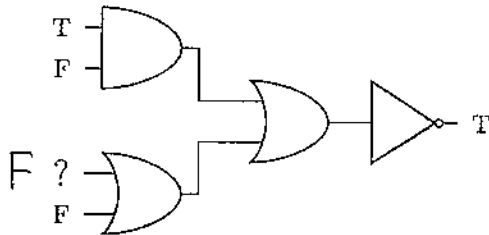
4 Logic Circuits

We can combine And, Or, and Not gates to make more complicated logic circuits.

1. What are the outputs of the following logic circuits?



2. Fill in the missing inputs on the following logic circuits.



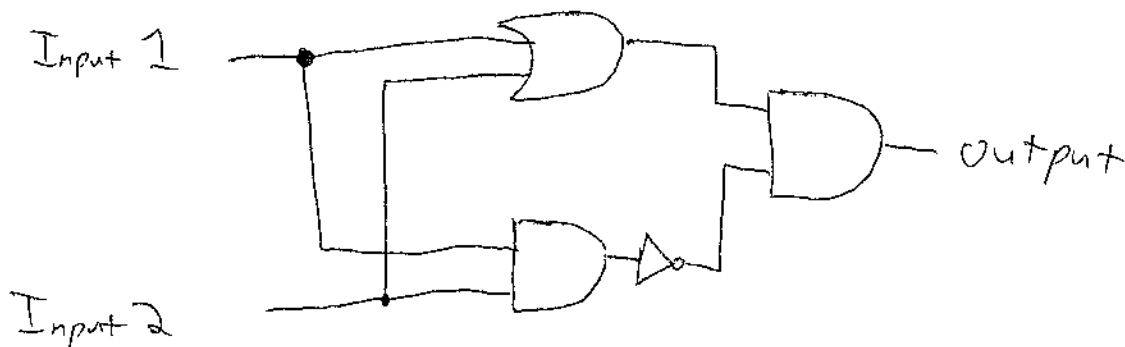
(In the last example, each input is fed into both an And gate and an Or gate.)

3. (*Challenge*) Design a logic circuit that has two inputs and one output, and obeys the following rules:

- The output is “True” if exactly one (not both) of the inputs is true.
- Otherwise, the output is “False”.

This logic circuit is called an “Exclusive Or gate”.

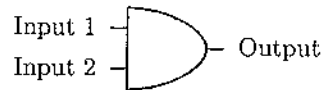
Hint: a single input can be fed into more than one logic gate.



You can think of this circuit as saying
“(A or B) and not (A and B).”

5 A “Universal” logic gate

1. Complete the table below, which completely describes the behavior of an And gate.



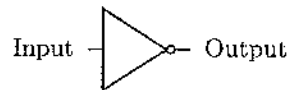
Input 1	Input 2	Output
F	F	? F (false)
F	T	? F (false)
T	F	? F (false)
T	T	? T (true)

2. Complete the table below, which completely describes the behavior of an Or gate.



Input 1	Input 2	Output
F	F	? F
F	T	? T
T	F	? T
T	T	? T

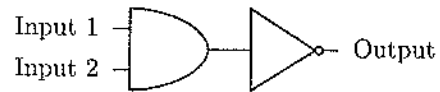
3. Complete the table below, which completely describes the behavior of a Not gate.



Input	Output
F	? T
T	? F

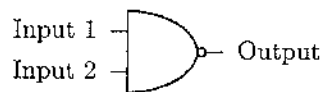
4. Design your own logic circuit in the space below. You can use And, Or, and Not gates. Make up inputs and figure out the output or outputs.

The logic circuit shown below is called a “Nand” gate:



“Nand” is short for “Not - And”. A “Nand” gate consists of an And gate followed by a Not gate.

To save writing in the future, there is a special symbol for a Nand gate:

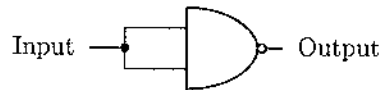


5. Complete the table below, which completely describes the behavior of a Nand gate.

Input 1	Input 2	Output
F	F	? T
F	T	? T
T	F	? T
T	T	? F

In the next few exercises, our goal is to show that any logic circuit can be made using only nand gates.

6. The logic circuit below has one input and one output:



(The Input value is fed into *both* inputs of the Nand gate.)

Fill in the table below, which describes the behavior of this logic circuit:

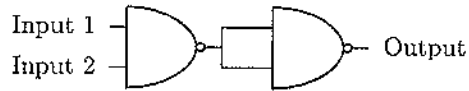
Input	Output
F	? T
T	? F

Compare this table with the table above that describes a Not gate. What do you notice?

The tables are identical.

Conclusion: A Not gate can be made out of a Nand gate !

7. Complete the table below, which describes the behavior of this logic circuit:



(The output of the first Nand gate is fed into *both* inputs of the second Nand gate.)

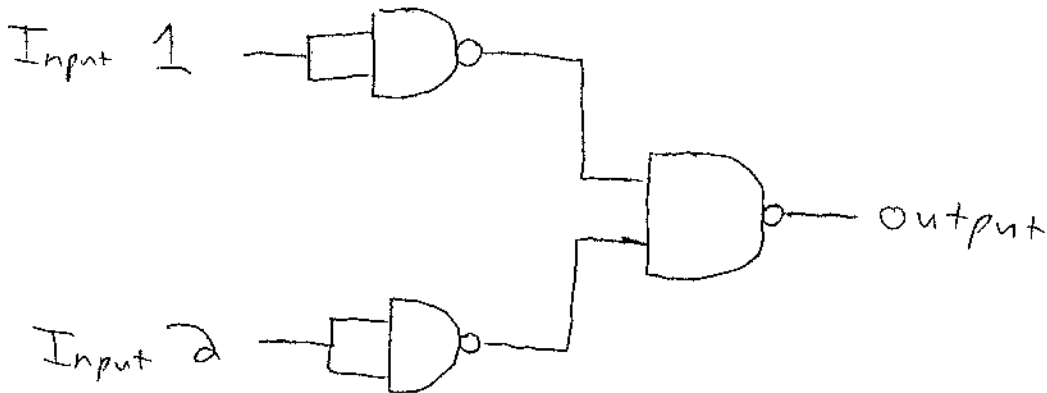
Input 1	Input 2	Output
F	F	? F
F	T	? F
T	F	? F
T	T	? T

Compare this table with the table describing an And gate. What do you notice?

They are identical.

Conclusion: An And gate can be made out of Nand gates!

8. (Challenge) How can you make an Or gate using only Nand gates?



You can think of this circuit as saying

"Not [{ Not (A and A) } and { Not (B and B) }]"

i.e., "Not [{ Not A } and { Not B }]"

We have shown that And, Or, and Not gates can be made using only Nand gates.

This means that any logic circuit whatsoever can be made using only Nand gates!

For this reason, the Nand gate is said to be "Universal".