

# LAMC Advanced Circle

January 29, 2017

Please print your name, first and last, followed by your school grade in the space below.

First Name

Last Name

Grade

In computational problems, answers not supported by computations yield no credit!

## Test

Pr #	1	2	3	4	5	6
Score	$\overline{5}$	$\overline{7}$	$\overline{7}$	$\overline{10}$	$\overline{7}$	$\overline{7}$

7	8	9	10	11	12	Total
$\overline{7}$	$\overline{7}$	$\overline{7}$	$\overline{7}$	$\overline{14}$	$\overline{15}$	$\overline{100}$

**Problem 1**

**5 pts**

*By the year 2050, Hyperloop has developed the following routes: Portland – New York City, New York City – Boston, Boston – Los Angeles, Los Angeles – San Francisco, San Francisco – New York City, San Francisco – Portland, San Diego – Washington DC, San Diego – Austin, Austin – Charlotte, Charlotte – Washington DC, Austin – Washington DC, and Los Angeles – New York City. Is it possible to get from Los Angeles to Washington DC by a Hyperloop train? Why or why not?*

**Problem 2****7 pts**

*In a small European country, each city is connected to other cities of the country by five roads. There are 25 inter-city roads in the country. How many cities are there?*

**Problem 3****7 pts**

*Is it possible to draw an octahedron without lifting the pencil off the paper? If you think it is possible, please draw the solid this way, indicating the order in which the edges are drawn by numbers. If you think it is not possible, please explain why.*

**Problem 4**

**10 pts**

*What is the minimal number of times you need to lift the pencil off the paper to draw a cube without repeating any edge? Give a reason for choosing the number.*

**Problem 5**

**7 pts**

*There are three ponds in a botanical garden, connected by ten non-intersecting brooks so that the ducks can swim from any pond to any other. How many islands are there in the garden?*

**Problem 6****7 pts**

*Draw a spherical projection of a spherical cube such that the North pole of the sphere is an interior point of one of the edges of the original graph.*

**Problem 7****7 pts**

*Let a simple connected planar graph have  $V$  vertices,  $E$  edges, and  $F$  faces. Prove that*

$$\sum \deg(F_i) = 2E.$$

**Problem 8****7 pts**

*Let a finite connected simple planar graph have  $E > 1$  edges and  $F$  faces. Prove that then  $2E \geq 3F$ .*

**Problem 9****7 pts**

*Prove that for a finite connected simple planar graph,  $E \leq 3V - 6$ .*

**Problem 10****7 pts**

*Prove that a connected simple planar graph contains at least one vertex of degree 5 or less.*

Recall that a *Platonic solid* is a 3D regular convex polyhedron. This means that all of its faces are congruent regular convex polygons, all the vertices have the same degree,  $D_v$ , and all of its faces have the same degree,  $D_f$ .

**Problem 11**

**14 pts**

*Prove that  $3 \leq D_v \leq 5$  and  $3 \leq D_f \leq 5$ .*

**Problem 12**

**15 pts**

*Classify Platonic solids.*



