

Math Circle  
Intermediate Group  
January 29, 2017  
Divisibility

**Warm-up problems**

1. Is  $2^9 \cdot 3$  divisible by

(a) 9?

(b) 8?

(c) 6?

2. Is it true that if a natural number is divisible by 4 and 6, then it must be divisible by 24?

1. How many zeros are there at the end of the decimal representation of the number  $100!$ ?

2. Tom multiplied two two-digit numbers on the blackboard. Then he changed all the digits to letters. Different digits were changed to different letters, and equal digits were changed to the same letter. He obtained  $AB \cdot CD = EEFF$ . Prove that Tom made a mistake in the multiplication.

3. Prove that the number  $(n^3 + 2n)$  is divisible by 3 for any natural number  $n$ .

4. Find the last digit of the number  $2^{50}$ .

5. Prove that  $2222^{5555} + 5555^{2222}$  is divisible by 7.

6. Find the smallest natural number which has remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, and a remainder of 5 when divided by 6.