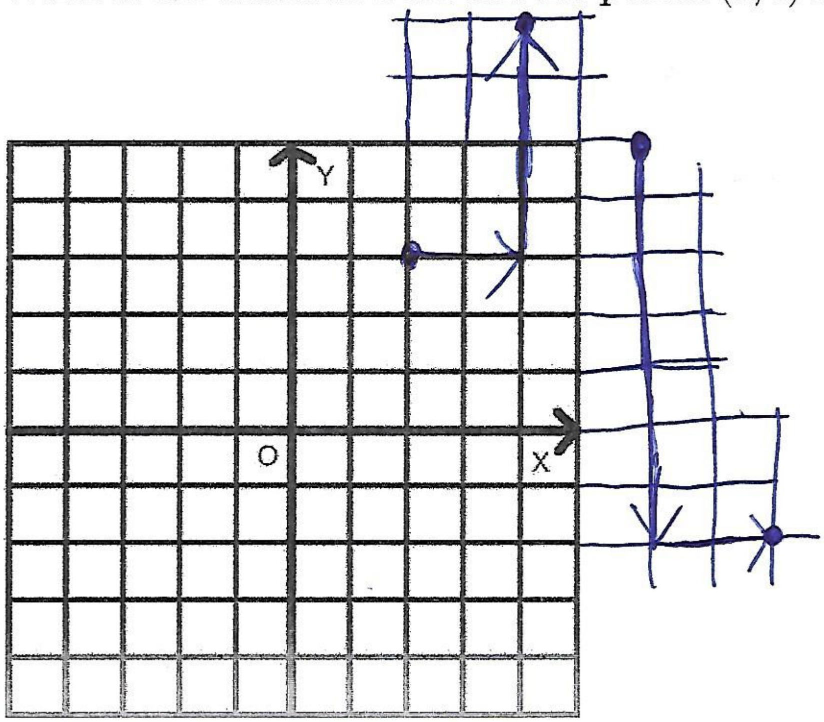
EXPLORING THE CITY OF DESCARTES - PART II

(1) The streets in the city of Descartes run in the north-south (up-down) and east-west (right-left) directions only. Bob the taxidriver measures distances between two houses in the city by finding the smallest number of blocks he has to travel (both vertically and horizontally) to reach one of the houses from the other in the fastest possible way.

(a) What is the distance between the points $(2, 3)$ and $(4, 7)$ according to Bob?



$\boxed{6}$

$(4-2)_x + (7-3)_y$

$= 2+4=6$

(b) What is the distance between the points $(6, 5)$ and $(8, -2)$ according to Bob?

$(8-6)_x + (5-{-2})_y = 2+7$

$= \boxed{9}$

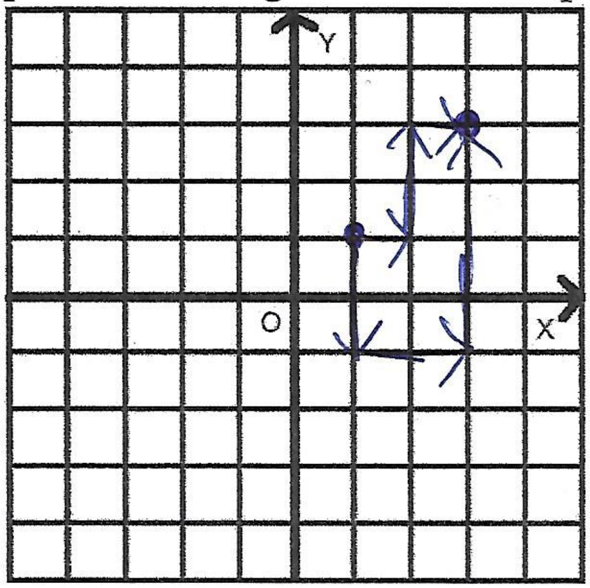
(2) Bob's passenger wants to go from $(1,1)$ to $(3,3)$ as follows:

- 2 blocks down;
- 2 blocks to the right;
- 4 blocks up;

This can be described by the following code:

$2\downarrow, 2\rightarrow, 4\uparrow$.

(a) Will Bob's passenger get to $(3,3)$ if he starts at $(1,1)$ and follows this plan? Use the grid below to help you:



He will make it to (3,3) following this road.

(b) How many blocks total does this route take?

$\boxed{8}$ $2+2+4=8$

(c) Bob says that he can get from $(1,1)$ to $(3,3)$ by driving only 4 blocks. Can you draw a possible route? Use a different colored pencil to draw a shorter route then describe it using the code above.

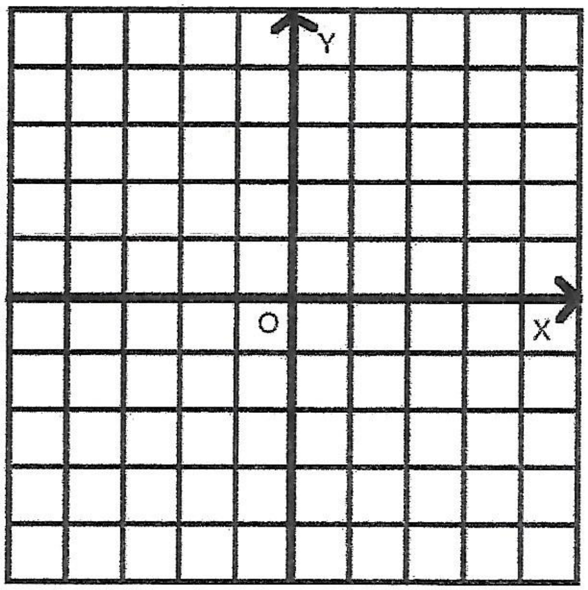
See the graph

(d) Can you find a route which is shorter than 4 blocks?

No

(e) What is Bob's distance from $(1,1)$ to $(3,3)$?

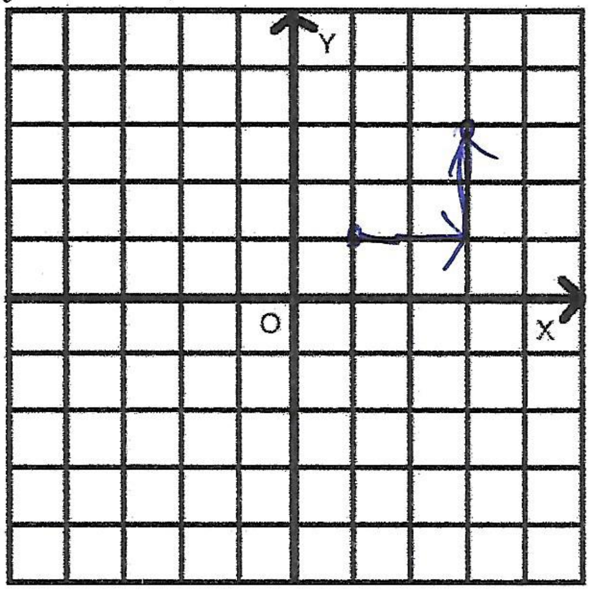
$\boxed{4}$



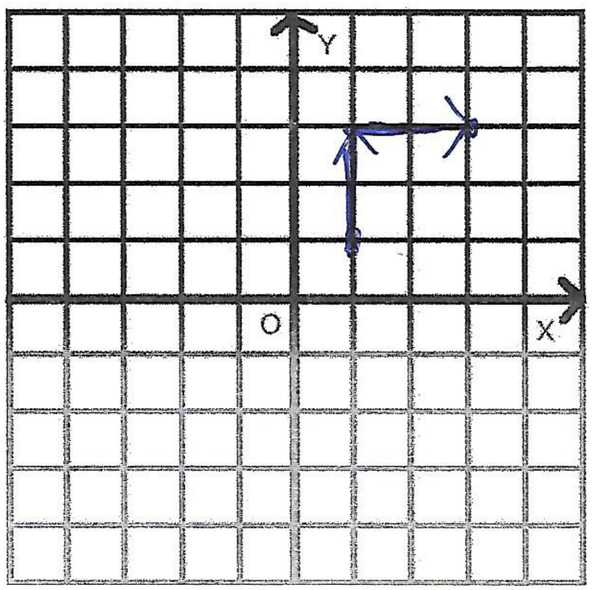
Can be used for

2) e)

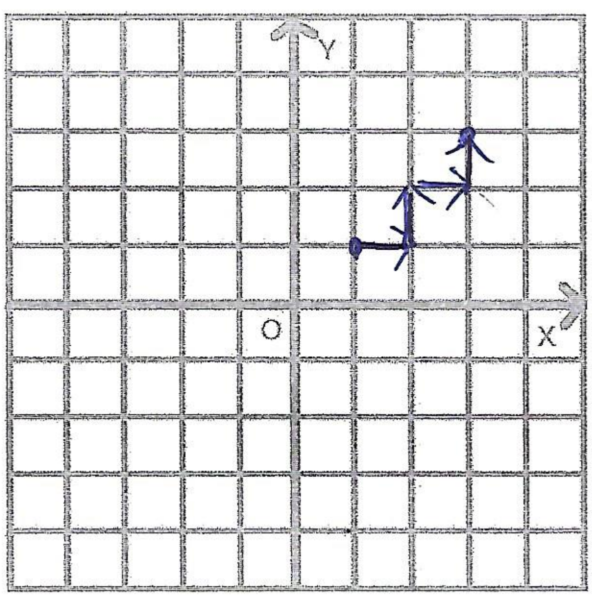
(3) Find all possible routes from $(1, 1)$ to $(3, 3)$ that have length 4. Draw each of the routes and describe it by a code: (*Note:* Write the code next to the route you drew)



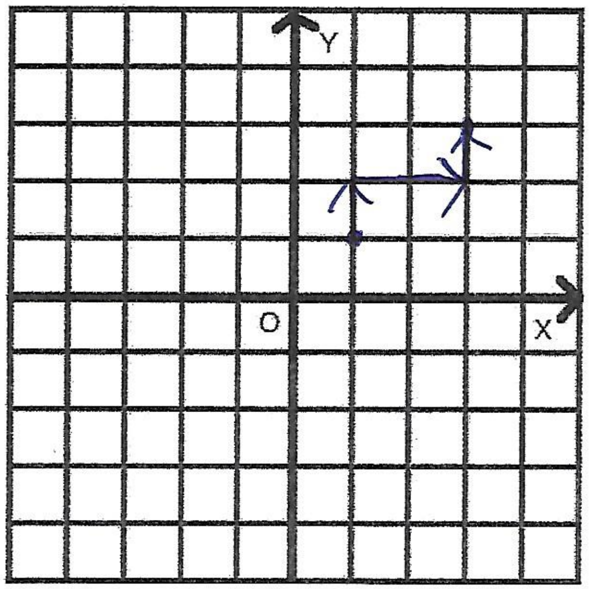
CODE: $2 \rightarrow \; 2 \uparrow$



CODE: $2 \uparrow \; 2 \rightarrow$



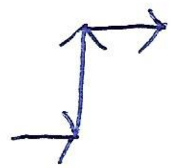
CODE: $1 \rightarrow 1 \uparrow 1 \rightarrow 1 \uparrow$



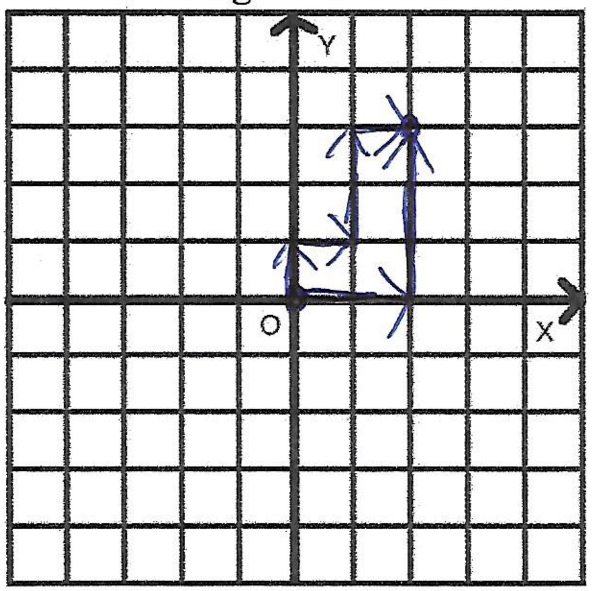
CODE: 1↑ 2→ 1↑

- Are there any routes that are shorter than 4?

No, and there are two other routes not shown.

 and 

(4) What is the distance between the points $(0, 0)$ and $(2, 3)$ according to Bob? Draw two possible routes that Bob can take from $(0, 0)$ to $(2, 3)$. Do they have the same length? Remember that Bob selects the *shortest* routes.



$(2-0)_x + (3-0)_y$

$= 2+3 = \boxed{5 \text{ blocks}}$

$\boxed{\text{Yes, same length}}$

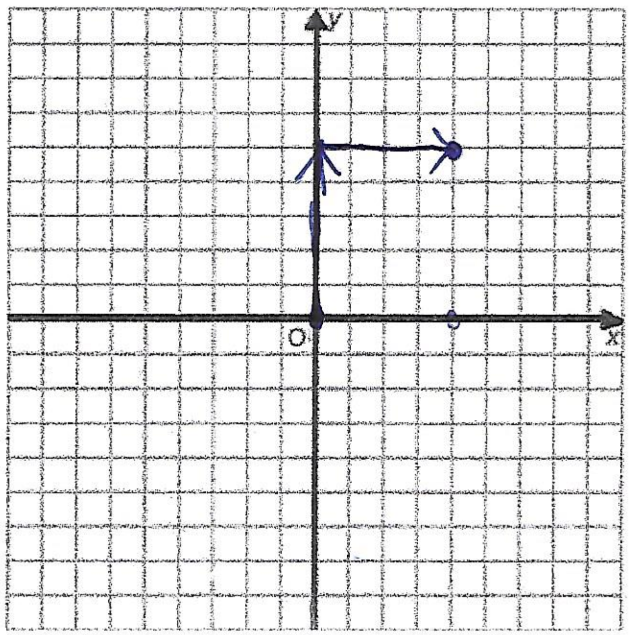
(5) Bob's passenger wants to go from $(0,0)$ to $(4,5)$. He thinks that the distance between these two points is 11 because when he drives himself he uses the following route:

$5 \rightarrow, 5 \uparrow, 1 \leftarrow.$

(a) Is he taking the shortest possible route?

NO

(b) Give an example of a shortest route. Draw the route and write down the code for it.



CODE: $5\uparrow 4 \rightarrow$

(6) The mayor of the city wants to go from $(0, 0)$ to $(1000, 1000)$. The city council proposed several routes that start as follows:

- $100 \uparrow, 53 \rightarrow, 27 \downarrow, 50 \rightarrow, 50 \uparrow$ ...
- $230 \leftarrow, 60 \uparrow, 80 \rightarrow, 330 \downarrow, 145 \rightarrow$ ...
- $500 \uparrow, 200 \rightarrow, 200 \uparrow, 300 \rightarrow, 300 \uparrow$ ...
- $400 \rightarrow, 200 \uparrow, 300 \rightarrow, 100 \downarrow, 350 \rightarrow$ ...
- $600 \uparrow, 145 \downarrow, 500 \rightarrow, 200 \leftarrow, 700 \uparrow$ ...

Assume the mayor wants to get there as fast as possible. What are the routes he should *not* take? Why?

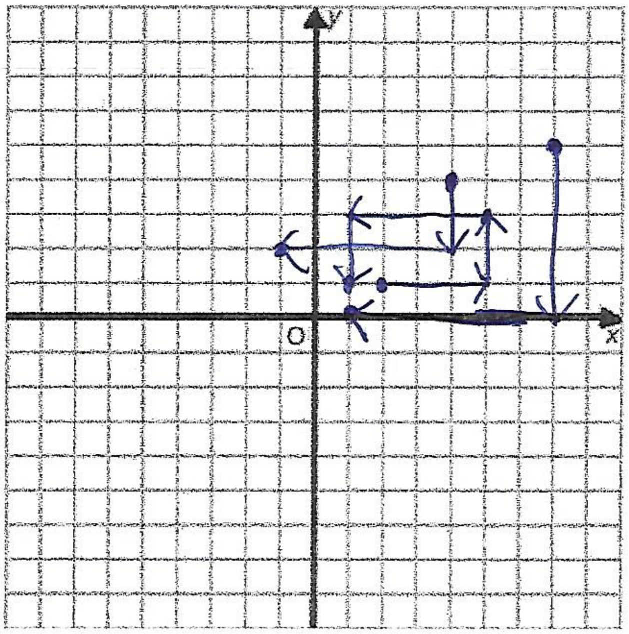
all of them except for the circled one, the others

Select the best route from the routes above and input the last segment.

go down and left when you only need to go right and up.

$\boxed{500 \rightarrow}$

(7) After working for a while, Bob decided to always go left (or right) as much as possible and then up (or down) as much as possible so his route consists of only two segments (horizontal, then vertical). Draw the routes that follow this idea between the following points and write the codes:



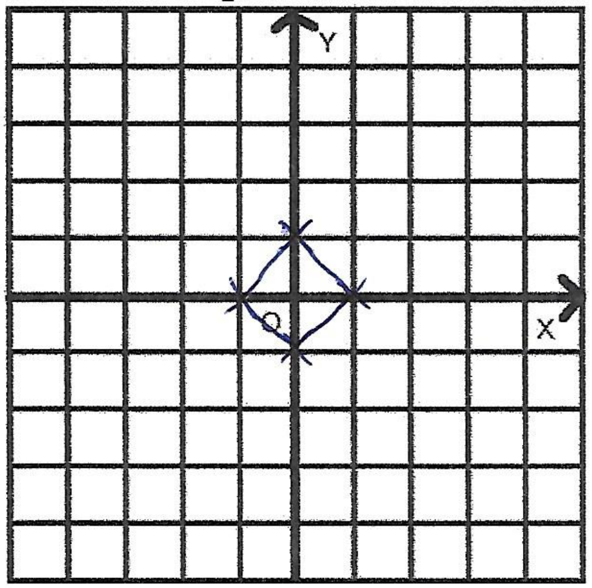
Starting Point	Ending point	Code	Bob's distance
$(3,3)$	$(4,6)$	$1\rightarrow, 3\uparrow$	4
$(4,4)$	$(-1,2)$	$5\leftarrow\ 2\downarrow$	7
$(2,1)$	$(5,3)$	$3\rightarrow\ 2\uparrow$	5
$(5,3)$	$(1,1)$	$4\leftarrow\ 2\downarrow$	6
$(7,5)$	$(1,0)$	$5\downarrow\ 6\leftarrow$	11
$(0,0)$	(a,b)	$a\rightarrow\ b\uparrow$	$a+b$

(a) If Bob's route goes horizontally and then vertically, how is Bob's distance expressed in terms of the numbers in the code?

The numbers in the code are simply the difference between the x and y coordinates frome the starting to end point.

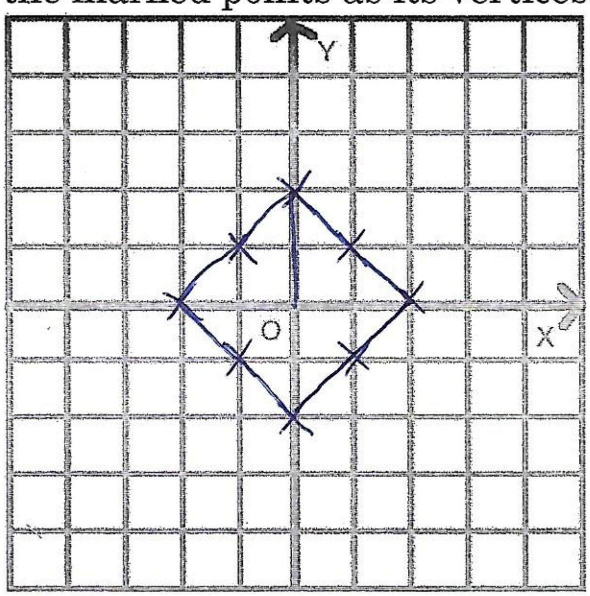
(8) Bob starts at $O = (0,0)$. He goes at the speed of one block per minute. (So it takes him 1 minute to go from $(0,0)$ to $(1,0)$, or to go from $(0,0)$ to $(0,1)$). Moreover, Bob *always drives away from O.*

(a) Mark all the points that Bob can reach in exactly 1 minute with an X, starting from O. Then draw lines as the perimeter of the shape that has the marked points as its vertices. How many such points are there?



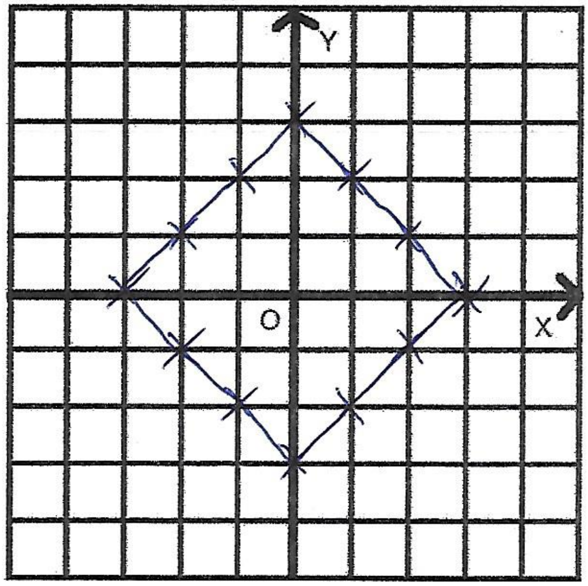
$\boxed{4}$

(b) Mark all the points that Bob can reach in exactly 2 minutes with an X, starting from O. Then draw lines as the perimeter of the shape that h[as] the marked points as its vertices. How many such points are there?



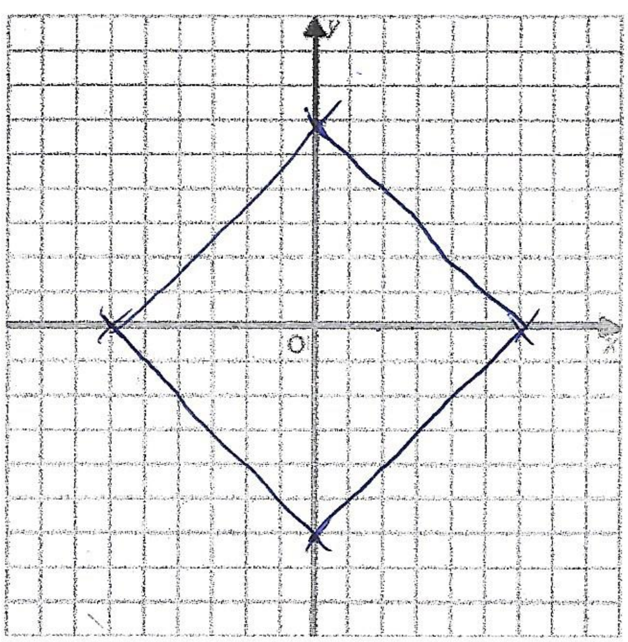
$\boxed{8}$

(c) Mark all the points that Bob can reach in exactly 3 minutes with an X, starting from O. Then draw lines as the perimeter of the shape that has the marked points as its vertices. How many such points are there?



$\boxed{12}$

(d) Describe the general pattern. What shape do you get when you draw lines as the perimeter of the shape that has its vertices at points that are a fixed taxicab distance from $(0,0)$? Can you explain this?



We get a square centered at Origin. This is because equal Taxicab distances moving around agrid ~~cause~~ form a diagonal line with slope 1, which causes a polygon with 4 equal sides: a square,

(9) Bob's friend Dan is a helicopter pilot. Dan can fly between any two points in the city along a straight line. So, Dan's distance is the usual distance between points.
We will compare distances between various points from Bob's and Dar point's of view:

(a) Give an example of a pair of points for which Bob's distance is the same as Dan's distance:

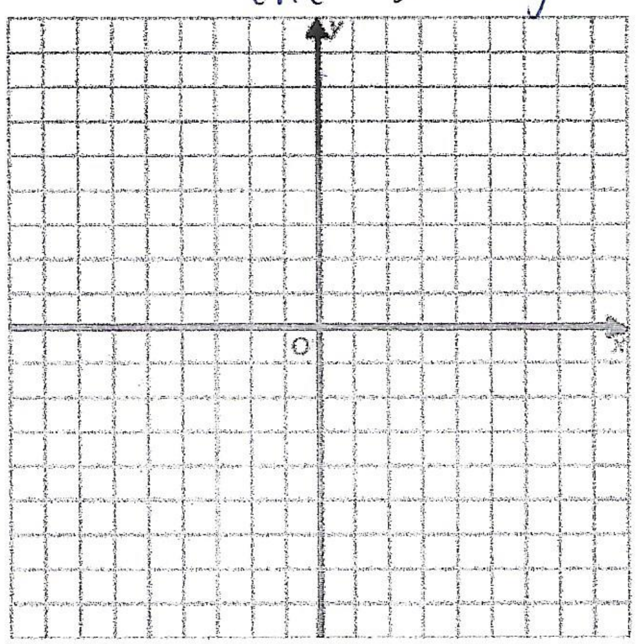
$(1,2) \to (1,5)$

(b) Give an example of a pair of points for which Dan's distance is shorter then Bob's distance:

$(1,1) \to (3,3)$

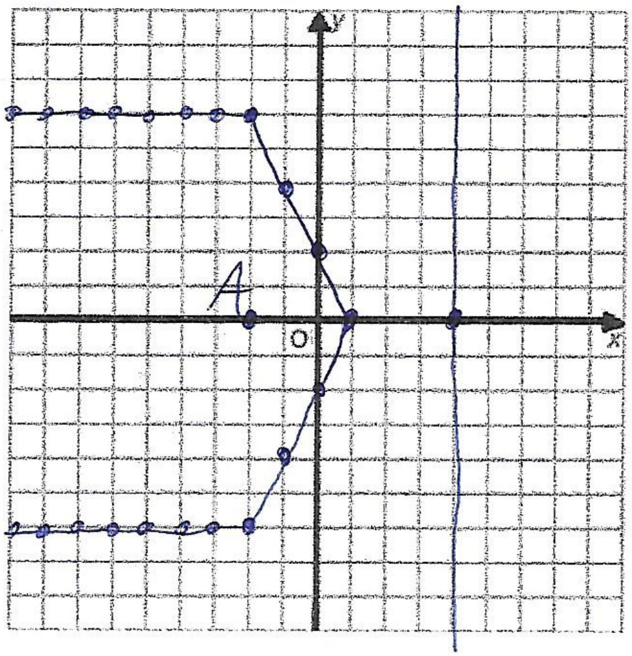
(c) Are there any pairs of points for which Bob's distance is smaller than Dan's distance? Why or why not?

NO! Dan will always fly over the straight line connecting the points, where Bob will at best do the same

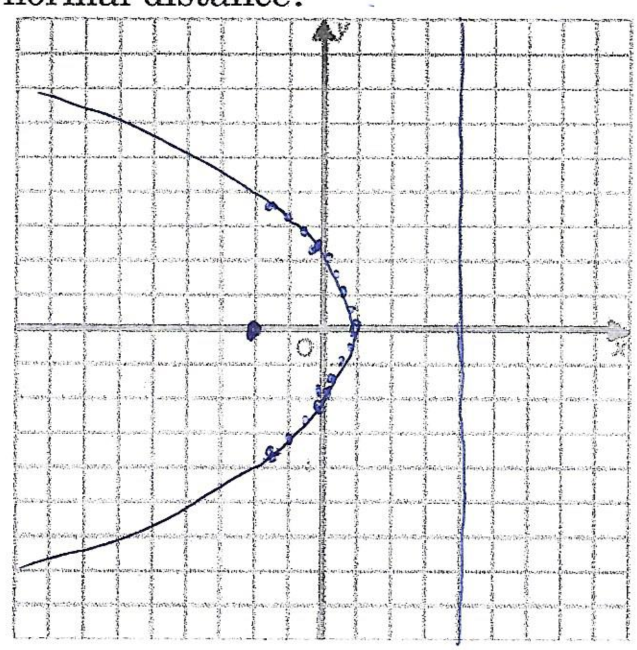


(10) Homework-Transmuting Euclidean Shapes

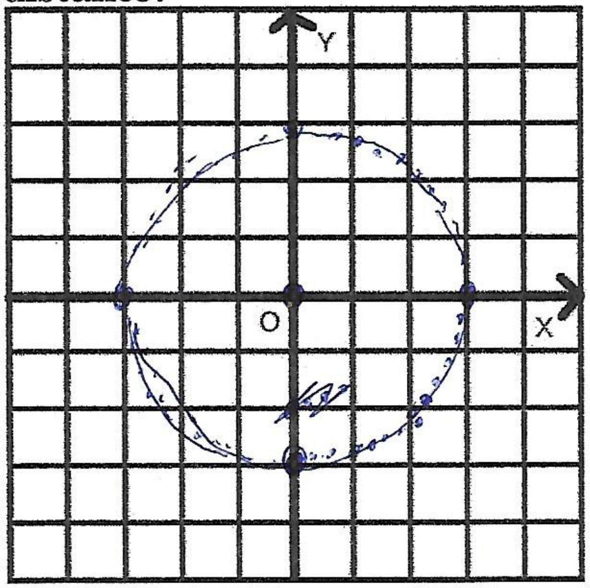
(a) Draw the vertical line at $x = 4$ and plot the point $A = (-2, 0)$. Now plot each point that is of equal Taxicab distance between the line as well as the point A.



(b) This is a Taxicab transmutation of the Euclidean Parabola. Point A is called its focus and the line is called its directrix. What would this shape look like on the graph below if we are not using Taxicab distance but normal distance?



(c) In problem 8, we drew a shape that has its points of equal Taxicab distance from a given center. What would this shape look like using normal distance?



A CIRCLE!