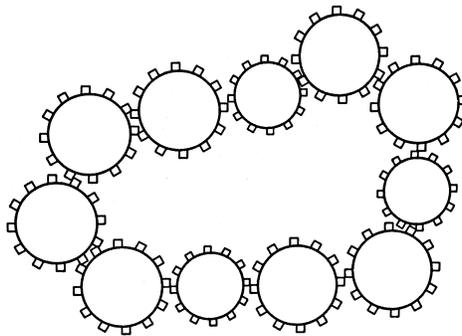


Math Circle  
Intermediate Group  
January 22, 2017  
Parity

**Warm-up problem**

Eleven gears are placed on a plane, arranged in a chain as shown below. Can all gears rotate simultaneously?



## Parity

An even number is said to have even parity, and an odd number has odd parity.

1. Can a knight start at square  $a1$  of a chessboard and go to square  $h8$ , visiting each of the squares on its way exactly once?

2. Pete bought a notebook containing 96 sheets and numbered them from 1 through 192. Victor tore out 25 sheets of Pete's notebook and added the 50 numbers he found on the pages. Could Victor have gotten 1990 as the sum?

3. There are 100 soldiers in a detachment, and every evening three of them are on duty. Can it happen that after a certain period of time, every soldier has shared duty with every other soldier exactly once?

4. The numbers  $1, 2, 3, \dots, 2016, 2017$  are written on a blackboard. We decide to erase from the blackboard any two numbers and replace them with their positive difference. After this is done several times, a single number remains on the blackboard. Can this number equal 0?

5. Can a convex 13-gon be divided into parallelograms?

6. A 17-digit number is chosen, and its digits are reversed, forming a new number. These two numbers are added together. Show their sum contains at least one even digit.

7. Nine numbers are placed around a circle: four 1s and five 0s. The following operation is performed on the numbers: between each adjacent pair of numbers is placed a 0 if the numbers are different and a 1 if the numbers are the same. The old numbers are then erased. After several of these operations, can all the remaining numbers be equal?

8. Is it possible to arrange the numbers from 1 through 9 in a sequence so that there are oddly many numbers between 1 and 2, between 2 and 3, ... , and between 8 and 9?