Exploring the City of Descartes Part 1

January 21, 2017

Part I

Review of Coordinates on a Graph

1. For a point (x, y) on the plane, the first number is called x-coordinate and the second one is called the y-coordinate.
   For example, for point $P = (-1, 4)$ we have $x = -1$; $y = 4$.

   (a) Find the x-coordinate of the following points:

   i. $A = (3, 2) \Rightarrow x =$;

   ii. $B = (4, -5) \Rightarrow x =$;

   iii. $C = (-7, -9) \Rightarrow x =$;

   (b) Find the y-coordinate of the following points:

   i. $A = (3, 2) \Rightarrow y =$;

   ii. $B = (4, -5) \Rightarrow y =$;

   iii. $C = (-7, -9) \Rightarrow y =$;

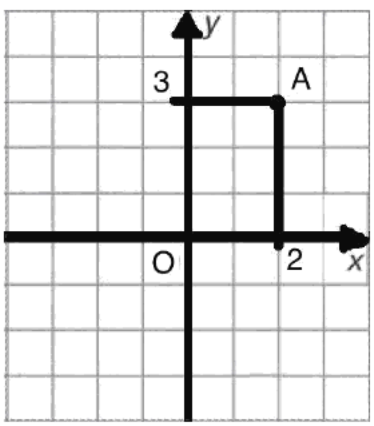
Part II

The City of Descartes

René Descartes was a French mathematician, philosopher and writer. Among his many accomplishments, he developed a very convenient way to describe positions of points on a plane. This method was very important for future development of mathematics and physics. We will start learning about this invention today.

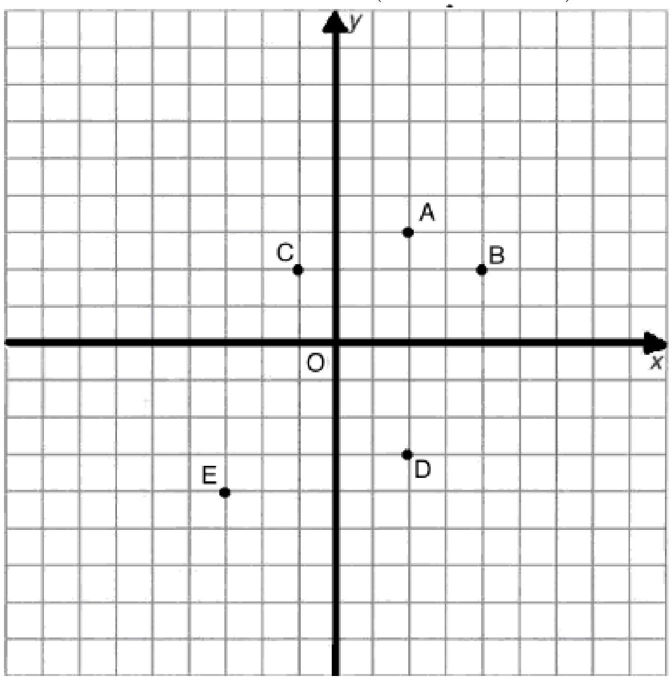
The city of Descartes is a plane that extends infinitely in all directions:

- The center of the city is marked by point O.
- The horizontal (West-East) line going through O is called the x-axis.
- The vertical (South-North) line going through O is called the y-axis.
- Each house in the city is represented by a point which is the intersection of a vertical and a horizontal line. Each house has an address which consists of two whole numbers written inside of parenthesis. For example, $(2, 3)$ is an address of the house as shown below:



- The first number tells you the x-value. However, it is also the distance to the y-axis. The distance is *positive* if you are on the right of the y-axis. The distance is *negative* if you are on the left side of the y-axis.
- The second number tells you the y-value. However, it also is the distance to the x-axis. The distance is *positive* if you are above the x-axis. The distance is *negative* if you are below the y-axis.

1. Let's find the addresses(coordinates) of several points in the city:



(a) Point O has address (,);

(b) Point A has address (,);

(c) Point B has address (,);

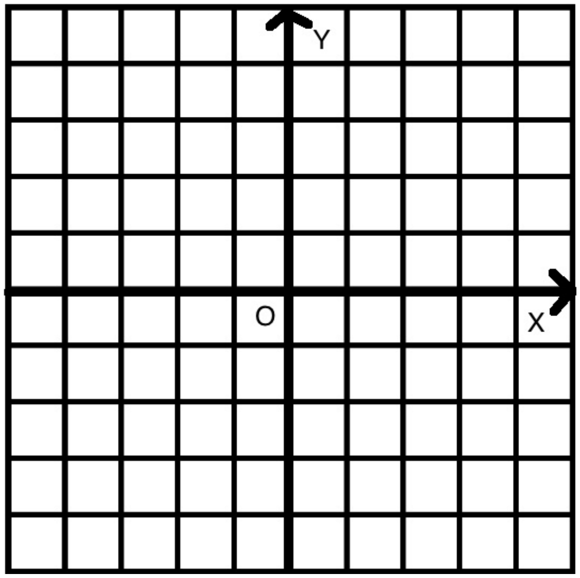
(d) Point C has address (,);

(e) Point D has address (,);

(f) Point E has address (,);

(g) The midpoint between A and D has address (,);
(Hint: The midpoint is the point on segment AD which is the same distance to A as it is to D. You can think of it as the "middle".)

2. Let's plot several points whose coordinates are given:



(a) Plot point F with address $(1, 4)$;

(b) Plot point G with address $(4, 1)$;

(c) Plot point H with address $(5, 3)$;

(d) Plot point J with address $(2, 5)$;

(e) Plot point K with address $(0, 2)$;

(f) Plot point L with address $(3, 0)$.

3. Let n be any whole number. Describe where the points with the following addresses are located:

(a) with addresses $(n, 0)$:

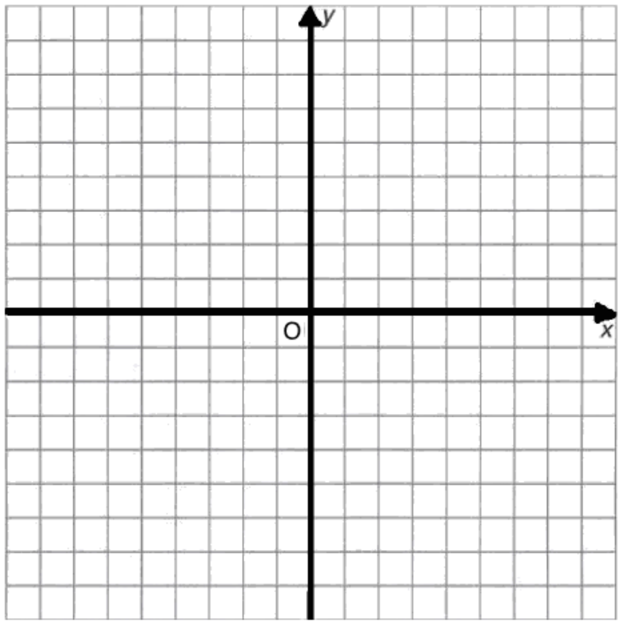
(b) with addresses $(0, n)$:

(c) with addresses $(n, 5)$:

(d) with addresses $(5, n)$:

(e) with addresses (n, n):

4. Plot the points and find the distance between the following points:



(a) $(4, 3)$ and $(4, 7)$;

Distance=

(b) $(-1, 3)$ and $(-1, 5)$;

Distance=

(c) $(6, 5)$ and $(8, 5)$;

Distance=

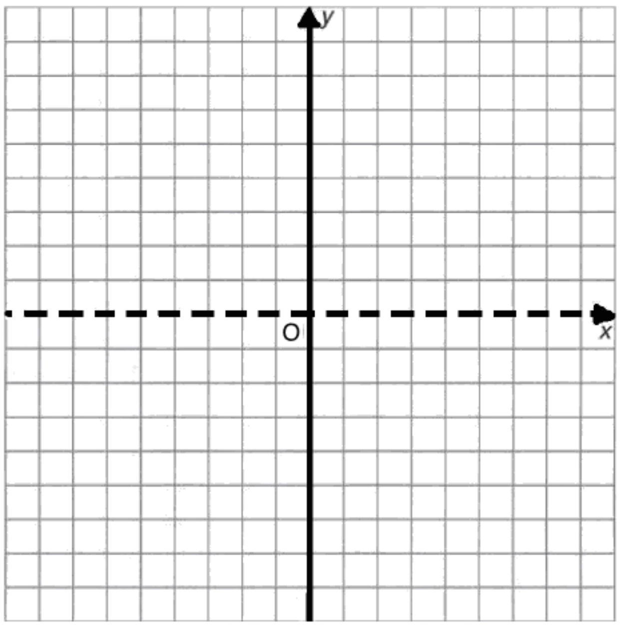
(d) $(5, -2)$ and $(7, -2)$;

Distance=

Part III

Reflections, Directions, and Squares

1. Now imagine that the x-axis is a mirror. Reflect each of the points below on the graph and find the address of the reflection:



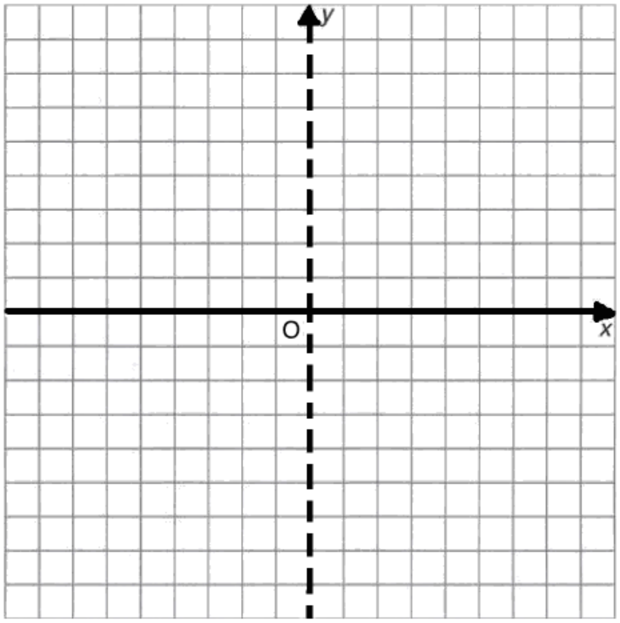
(a) $(2,3)$ Reflection point: (,)

(b) $(5,1)$ Reflection point: (,)

(c) $(-6,2)$ Reflection point: (,)

(d) $(-3,4)$ Reflection point: (,)

2. Plot the same starting points as in the last problem. Now imagine that the y-axis is a mirror. Reflect the points below and find the address of the reflection of each point:



(a) $(2, 3)$ Reflection point: (,)

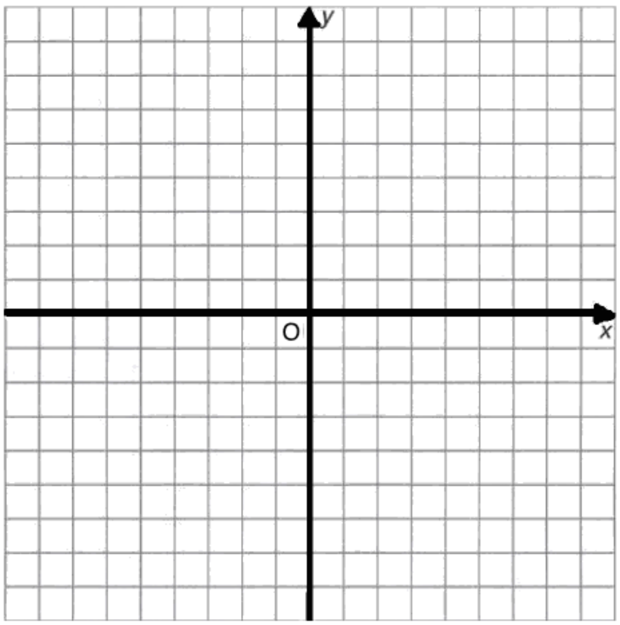
(b) $(5, 1)$ Reflection point: (,)

(c) $(-6, 2)$ Reflection point: (,)

(d) $(-3, 4)$ Reflection point: (,)

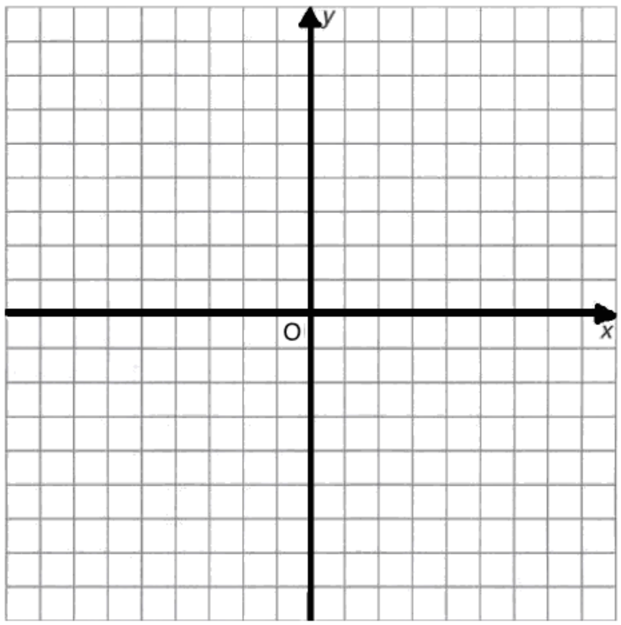
3. Mark each step below on the graph. You start at the point $(0, 0)$. Then

- Go north (up) for two units;
- Go east (right) for 5 units;
- Go south for 1 unit;
- Go west for 1 unit.

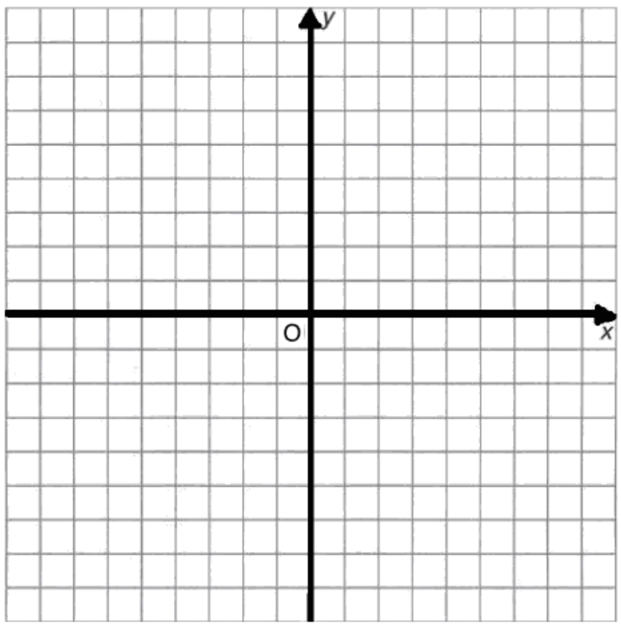


Where are you now? Give the address of the final point.

4. Two of the vertices of a square are at points $(2, 3)$ and $(5, 3)$. Find the other two vertices for the square and write down their coordinates. *Note:* The sides of the squares are vertical and horizontal. *Hint:* There is more than one solution, find all possible solutions.



5. Two of the vertices of a square are at the points $(0, 0)$ and $(4, 4)$. Two of the sides of the square are vertical and the other two sides are horizontal. Find the other vertices of this square:



Part IV

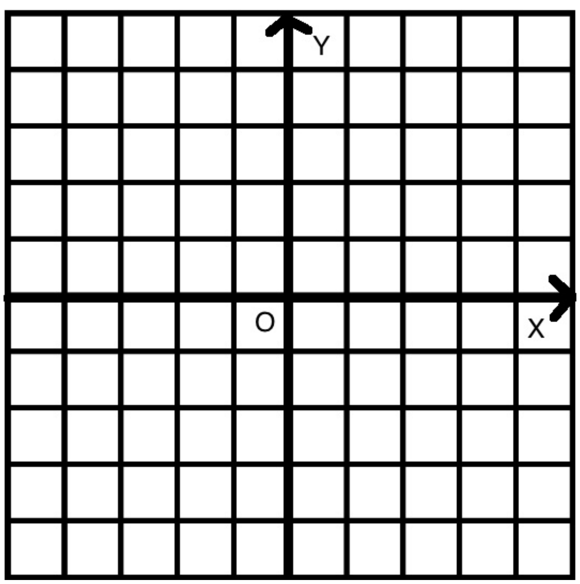
Finishing Up Coordinates

1. The houses of Amy (A), Ben (B), Cindy (C), and Dan (D) are vertices of a square:

- The center of this square is at the point $O = (0, 0)$;
  - The length of each of the sides of this square equals to 4;
  - Amy's house is directly to the north from Dan's house;
  - Ben's house is east from Amy's house;

  Denote the vertices of this square by A, B, C, D. Mark the houses (vertices) on the picture below and find their addresses:

$$\begin{aligned} A &= (\quad , \quad), \\ B &= (\quad , \quad), \\ C &= (\quad , \quad), \\ D &= (\quad , \quad), . \end{aligned}$$

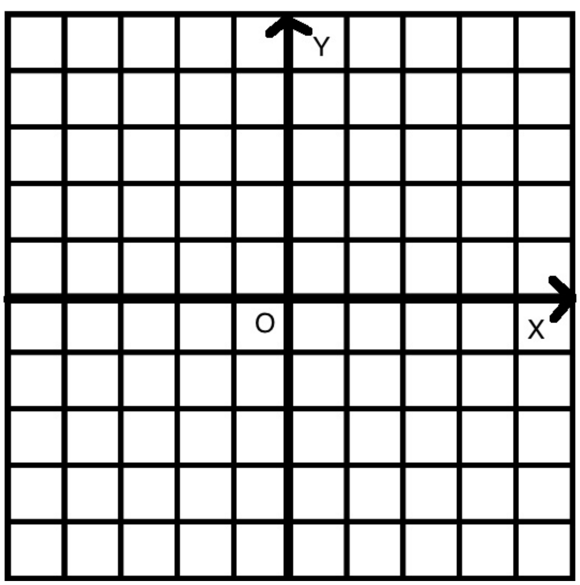


1. The houses of Eddie (E), Fred (F), George (G), and Helen (H) are also vertices a square:

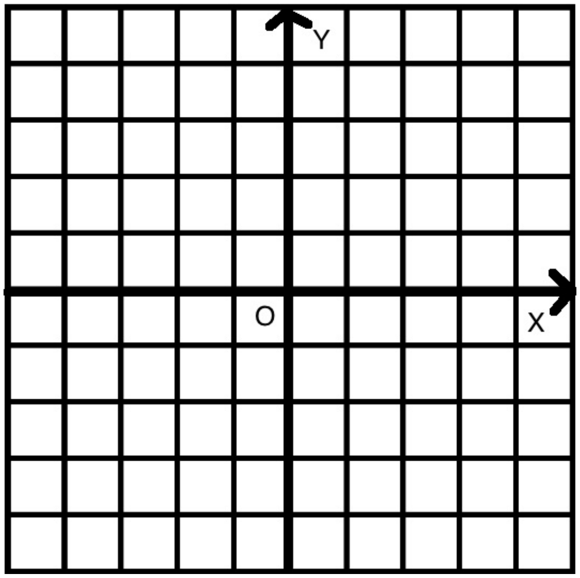
   - The center of this square is at the point $O = (0, 0)$;
   - The distance from O to any of these houses is 2;
   - George's house is east of Eddie's house;
   - Fred's house is north of Helen's house;
   - Denote the vertices of this square by E, F, G, H.

   Mark the vertices on the same picture and find their addresses:

$$
\begin{aligned}
E &= (\quad , \quad), \\
F &= (\quad , \quad), \\
G &= (\quad , \quad), \\
H &= (\quad , \quad), .
\end{aligned}
$$



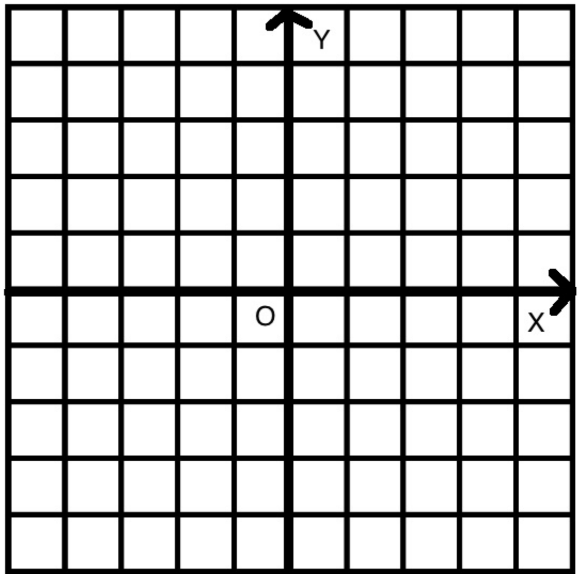
2. Plot several points and reflect them across the x-axis. (Imagine that x-axis is a mirror)



(a) Does the x-coordinate change when you reflect a point across the x-axis? If so, how?

(b) Does the y-coordinate change when you reflect a point across the x-axis? If so, how?

3. Plot several points and reflect them across the y-axis. (Imagine that y-axis is a mirror)



(a) Does the x-coordinate change when you reflect it across the y-axis? If so, how?

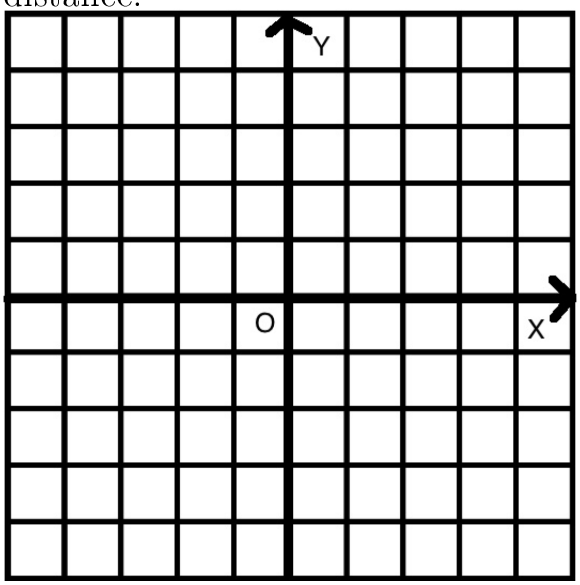
(b) Does the y-coordinate change when you reflect it across the y-axis? If so, how?

Part V

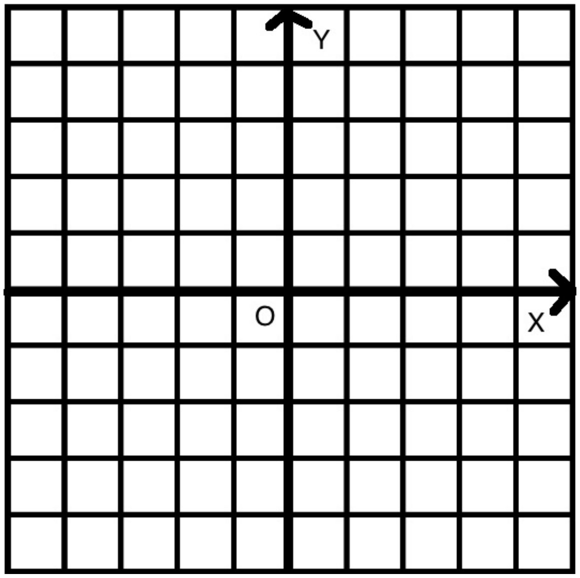
Introduction to Taxicab Geometry

1. Let's briefly introduce the concept of Taxicab Geometry for next week. We have been calling the coordinate plane Descartes's "city" because we want you to think of it as a city with streets along the lines of the graph and city blocks in the squares of the graph. In a city, the shortest path between two points is not necessarily a straight line because you always have to go along the streets. The **Taxicab distance** between them is the shortest path bewteen them that goes along the "streets" of the city.

   Plot the coordinates $(3, -2)$ and $(-4, 1)$ on the graph below and find their Taxicab distance.



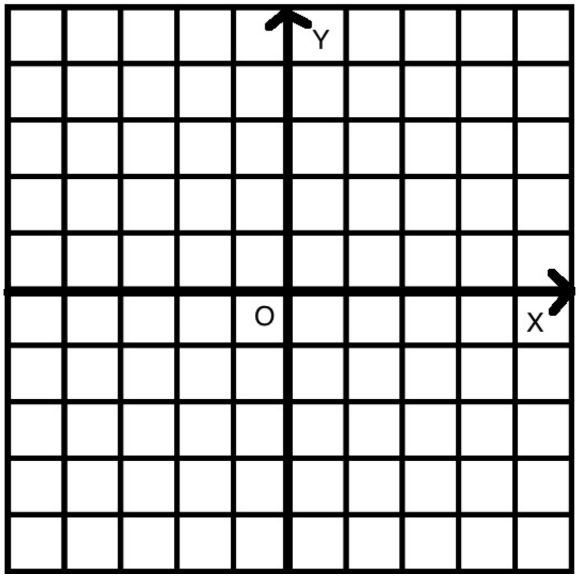
2. Now plot the points $A = (3, 3)$, $B = (1, 1)$, and $C = (-2, 1)$ on the graph below.



What is the Taxicab distance from A to B and from B to C? Is A or C closer to B? Which one seems closer to B if we draw straight lines from B to A and B to C?

Part VI
Homework

Plot these 3 points on the graph below: $A = (0, 3)$, $B = (4, -2)$, and $C = (-5, 0)$.



Find the point Q on the graph that is as close as possible to points A, B, and C by using Taxicab distances. More explicitly,
Find a point Q such that the sum of the Taxicab distances from Q to A, B, and C is at its minimum value:
$d_T(Q, A) + d_T(Q, B) + d_T(Q, C)$ is as small as possible where $d_T(x, y)$ is the Taxicab distance between the points x and y.