

WORKING TOGETHER

BEGINNERS 01/18/2015

- (1) Five rabbits eat five carrots in five minutes. How many rabbits will eat 10 carrots in 10 minutes?

Direct proportion:

$$r = \frac{(k)C}{m}$$

$$\frac{5 \times 5}{5} = \frac{r \times 10}{10}$$

$$r = 5$$

r = rabbits
 k = constant
 C = carrots
 m = minutes

1 rabbit eats 1 carrot = 5 minutes
1 rabbit eats 2 carrots = 10 minutes

- (2) Six boys wash six windows in 30 minutes. How many boys would it take to wash 48 windows in 3 hours?

$$b = \frac{kW}{m}$$

$$\frac{bm}{W} = k$$

$$\frac{6 \times 30}{6} = \frac{b \times (3 \times 60)}{48}$$

$$30 = \frac{b \times 60}{16}$$

$$b = \frac{30 \times 16}{60} = 8$$

$$b = 8 \text{ boys}$$

(3) John and Jane can clean the house in 4 hours and 30 minutes. Jane works three times as fast as John.

(a) What percentage of the work was done by John?

$$25\% \\ = \frac{1}{4}$$

(b) What percentage of the work was done by Jane?

$$75\% \\ = \frac{3}{4}$$

(c) How long would it take for John to clean the house by himself?

In 1 hour, $\frac{1}{4.5}$ work done by both

$$x + 3x = \frac{2}{9} \Rightarrow 4x = \frac{2}{9} \Rightarrow x = \frac{2}{36} \rightarrow \text{work done by John in 1 hour}$$

because Jane does 3 times the work as John

(d) How long would it take for Jane to clean the house by herself?

$$\Rightarrow \text{total work done in } \frac{36}{2} = 18 \text{ hours}$$

$$3x = \frac{6}{36} = \frac{1}{6} \rightarrow \text{work done by Jane in 1 hour}$$

$$\Rightarrow \text{total work done by Jane in 6 hours.}$$

- (4) John can mow the lawn in 60 minutes. His brother Peter can do it in 30 minutes. How fast can they do it if they work together?

John - $\frac{1}{60}$ of the lawn in 1 minute

Peter - $\frac{1}{30}$ of the lawn in 1 minute.

$$\begin{aligned} \text{In 1 minute, } & \frac{1}{60} + \frac{1}{30} \text{ of the lawn} \\ & = \frac{1+2}{60} = \frac{1}{20} \text{ of the lawn.} \end{aligned}$$

\Rightarrow 20 minutes to do the whole lawn.

- (5) Three teachers were grading an exam. If they would work by themselves, it would take them 8 hours to do all the work. However, 2 hours after they started, several colleagues joined them. In the end, all work was done in just 4 hours (since the moment the first three teachers started). How many colleagues joined the teachers?

3 teachers \rightarrow 8 hours

\Rightarrow In 1 hour, $\frac{1}{8}$ of the work done by 3 teachers

2 hours, $\frac{1}{4}$ of the work.

$\Rightarrow \frac{3}{4}$ of the work left - finished in 2 hours.

\Rightarrow In one hour, $\frac{3}{8}$ of the work done by $3+X$ teachers

Since $\frac{3}{4}$ of the work gets done now, we have 3 times the teachers (i.e. 9 teachers). Therefore, 6 teachers joined.

- (6) The wide pipe fills the pool in 3 hours. The narrow fills the pool in 9 hours. How long would it take for the two pipes to fill the pool if both are open at the same time?

Wide pipe

$\frac{1}{3}$ of the pool in 1 hour.

Narrow pipe

$\frac{1}{9}$ of the pool in 1 hour

In 1 hour, $\frac{1}{3} + \frac{1}{9}$ of the pool = $\frac{3+1}{9} = \frac{4}{9}$ of the pool.

$\Rightarrow \frac{9}{4}$ hours to fill the whole pool.

- (7) The first (narrow) pipe fills the pool in 15 hours. The second (wide) pipe fills the pool in 10 hours. How long does it take for the two pipes to fill the pool if both are used at the same time?

(a) What portion of the pool does the first pipe fill in 1 hr?

$$\frac{1}{15}$$

(b) What portion of the pool does the second pipe fill in 1hr?

$$\frac{1}{10}$$

(c) What portion of the pool do the two pipes fill in 1 hr?

$$\frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}$$

(d) How many hours does it take for the two pipes to fill the pool?

$$6 \text{ hours} \quad (\text{inverse of } 1/6)$$

(e) What percentage of the pool was filled by the narrow pipe?

$$\begin{aligned} \text{In 6 hours, first pipe fills } \frac{6}{15} &= \frac{2}{5} \\ &= 40\% \end{aligned}$$

(f) What percentage of the pool was filled by the wide pipe?

$$\begin{aligned} \text{In 6 hours, second pipe fills } \frac{6}{10} &= \frac{3}{5} \\ &= 60\% \end{aligned}$$

- (8) The first crew can repair a road in 9 days. The second crew can do the same work in 12 days. The first crew worked on the road for 3 days. After that, the second crew took over and finished the work. How long did the second crew have to work to finish the job?

In 1 day, first crew - $\frac{1}{9}$ of the road
 second crew - $\frac{1}{12}$ of the road

In 3 days, $\frac{1}{3}$ of the road finished.

$\frac{2}{3}$ of the road left.

$$\frac{\frac{2}{3}}{\frac{1}{12}} = \frac{2 \times 12}{3} = \underline{\underline{8 \text{ days}}}$$

- (9) There are two pipes, one supplies hot water and the other supplies cold water. The pipe with hot water can fill a tank in 23 minutes, while the pipe with cold water can fill the same tank in only 17 minutes. If you open the hot pipe first, how soon do you need to open the cold pipe so that by the end you have one and a half times more hot water than cold water?

$$100\% = 1.5x + x \quad x = \text{cold water}; 1.5x = \text{hot water}$$

$$1 = 2.5x \Rightarrow \frac{2}{5}x \quad x = \frac{2}{5} = 40\% \quad 1.5x = 60\% = \frac{3}{5}$$

Hot pipe
 $(23 \text{ min}) \left(\frac{3}{5}\right) = \frac{69}{5} \text{ min}$

Cold pipe
 $(17 \text{ min}) \left(\frac{2}{5}\right) = \frac{34}{5} \text{ min}$

Time when only hot pipe on = $\frac{69}{5} - \frac{34}{5} = \frac{35}{5} = \underline{\underline{7 \text{ min}}}$

- (10) If two people work together, it takes them 12 days to complete the job. If the first person does half the work and the second person does the other half, it takes them 25 days to complete their task. How long would it take for each?

TRIAL AND ERROR:

If 50% & 50%

It would've taken 24 days if they did half & half.

If 60% & 40%

12 days together

5 \Rightarrow 30 days and 20 days individually.

- (11) Thirty workers can complete a job in 12 days working 8 hours per day. If the job is to be completed in 10 days by twenty-four men, how many hours must they work per day?

No. of hours indirectly proportional to no. of days
and indirectly proportional to no. of men.

OR work takes 12×8
hours from each worker,
OR $30 \times 12 \times 8$ total effort

$$\Rightarrow x \times 24 \times 10 = 30 \times 12 \times 8$$

$$x = 12.$$

$$h = \frac{k}{md}$$

$$\Rightarrow 30 \times 12 \times 8 = h \times 10 \times 24$$

$$\Rightarrow h = \frac{30 \times 12 \times 8}{10 \times 24} = 12 \text{ hours per day.}$$

- (12) It takes a horse two days to eat a pile of hay. It takes a cow three days to eat a pile of hay, and it takes a lamb six days to eat a pile of hay. How long would it take them if they ate the pile of hay together. What percentage of the hay would the horse eat? The cow? The lamb?

On one day, horse eats $\frac{1}{2}$ of the hay

cow eats $\frac{1}{3}$

lamb eats $\frac{1}{6}$

$$\Rightarrow \text{On one day, } \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$$

\Rightarrow takes 1 day.

- (13) A pipe fills a pool in three hours. The drain empties it in five hours. How long would it take to fill the pool if you are filling it with the first pipe while the drain is left open?

On one hour, $\frac{1}{3}$ filled by pipe, $\frac{1}{5}$ drained by drain.

when together, $\frac{1}{3} - \frac{1}{5}$ in 1 hour.

$$= \frac{5-3}{15} = \frac{2}{15} \text{ filled in 1 hour.}$$

$$\Rightarrow \frac{15}{2} = 7.5 \text{ hours to fill it.}$$

- (14) There are two ropes. It takes each rope 1 hour to burn completely. However, the burning is not uniform (that is, a rope can first burn fast and then slow down). How can you measure 45 minutes using these two ropes? (*Hint: Think what you can do with the ropes*).



Light both ends of A; one end of B.
 A burns in 30 minutes \Rightarrow 30 min of B left.
 Then light other end of B.
 Burns in 15 minutes.

$$30 + 15 = 45$$

- (15) (An ancient problem) It takes a man 14 days to drink a barrel of water. If both he and his wife drink the water, it takes them 10 days to finish the barrel. How long would it take for the wife to finish the barrel by herself?

$\frac{1}{14}$ of the barrel drunk by man in 1 day.

$\frac{1}{x}$ of the barrel drunk by wife in 1 day.

$$\frac{1}{\frac{1}{14} + \frac{1}{x}} = 10 \Rightarrow \frac{14x}{14+x} = 10$$

$$\Rightarrow 14x = 140 + 10x$$

$$\Rightarrow 4x = 140$$

$$x = 35$$

- (16) (Math Kangaroo Problem) A complete set of dominoes contains 28 pieces. The pieces show every possible combination of two numbers of dots from 0 to 6 inclusive. How many dots are there altogether in a complete set of dominoes?

No. of 6s $\rightarrow 8$

No of 0s $\rightarrow 8$

No of dots \rightarrow

$$8 \times 6 + 8 \times 5 + 8 \times 4 + 8 \times 3 + 8 \times 2 + 8$$

$$= 48 + 40 + 32 + 24 + 16 + 8$$

$$= 98 + 56 + 24$$

$$= 168$$

- (17) (Math Kangaroo Problem) Adam and Tom are walking in the same direction around a circular table and counting chairs. They begin their counts with different chairs. Tom's twentieth chair is Adam's fourth chair, while Tom's tenth chair is Adam's forty sixth chair. How many chairs are there at the table?

Tom's 20th \rightarrow Adam's 4th

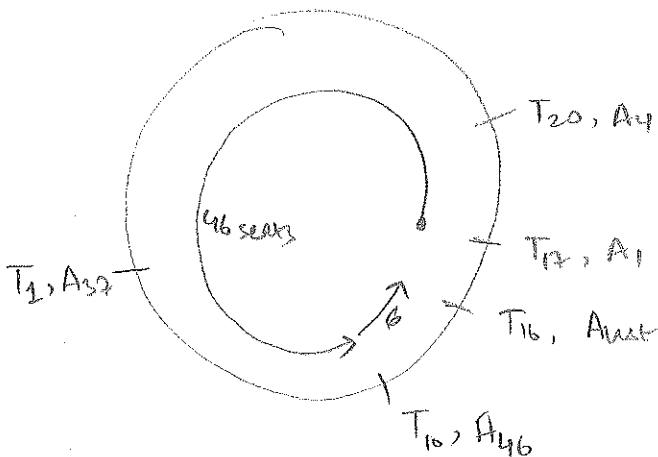
Adam's 1st \rightarrow Tom's 17th

\Rightarrow Adam's last \rightarrow Tom's 16th

Tom's 10th \rightarrow Adam's 46th

Tom's 1st \rightarrow Adam's 37th

Tom's last \rightarrow Adam's 36th



52