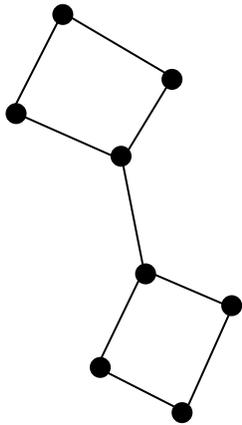


## Junior Circle Meeting 2 – Railroads and Trees

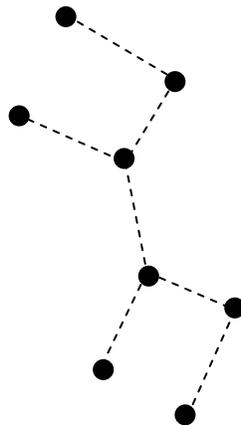
April 11, 2010

1. The insects of Antland now decide that in addition to tunnels which connect some of their cities together, they now want to build railroads! They decide that they want their railroads to be very efficient: they want there to be only ONE way to get to from any city to any other city. They also decide to only build railroads between cities that are directly connected by tunnels.

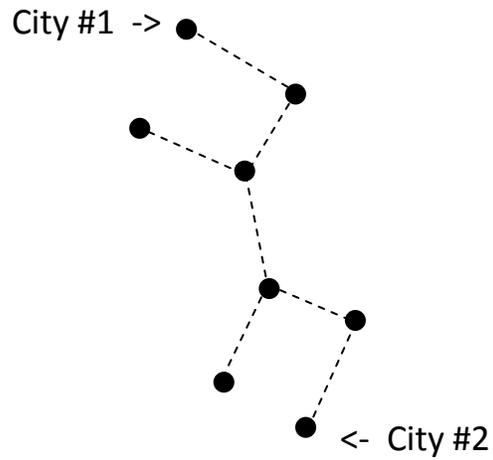
For example, one section of Antland looks like this:



Bob the Beetle has an idea of how to build railroads (so that there is only one way to get between any pair of cities). His idea is like this:



Let's now examine this further. Two cities have been labeled below. Can you draw the path connecting these two cities?



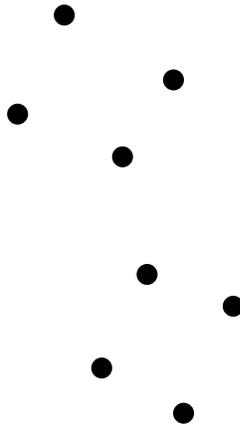
Is there another path connecting these two cities?

What about any other two cities? Is there only one path connecting any pair of cities? Try it!

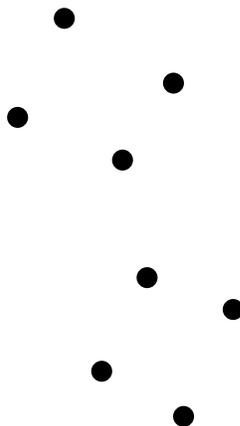
Was Bob the Beetle correct in thinking that this is a valid railroad system?

Now let's try to come up with our own railroad system. Remember: there must only be one railroad path between any two cities.

Draw a possible railroad path connecting the cities together below:

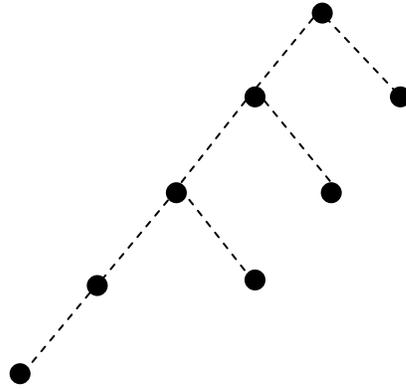


Now draw another one!

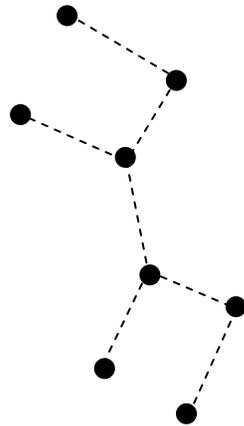


What is the same about these railroad systems?

2. In Beeland, the insects come up with a railroad system to connect their cities in the following way:



Bob the Beetle says that their world is exactly identical to Antland!!!  
Compare the two worlds and explain why Bob the Beetle is right!



### 3. Guess my number game

Cindy and Mindy are playing a game where one of the girls thinks of a number between 1 and 32 (inclusive) and the other girl tries to guess it.

a.) Can you think of a good strategy to guess a number, in the smallest number of guesses? The guessing player must ask “yes” or “no” questions.

b.) Cindy and Mindy decide a good strategy is to guess a number halfway and if the number is not correct, then the other person can either say “higher” or “lower” to help the guesser. If the number is correct, then the other person shouts, “correct!”

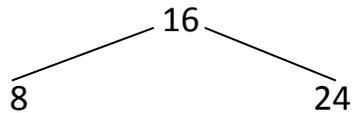
For example, let’s say Cindy thinks of the number 4. Mindy starts by guessing 16, since 16 is halfway between 1 and 32. Then Cindy will say “lower”. Then Mindy will guess 8, and Cindy will say “lower”. Then Mindy will guess 4 and Cindy says “correct!” and the game is done!

Find a partner and try this for yourself!

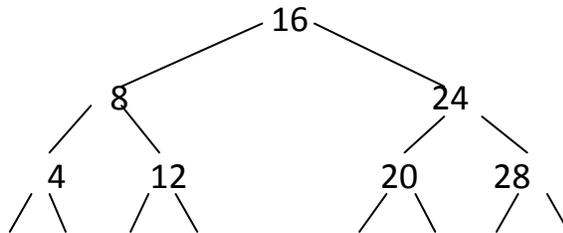
#### 4. Trees

- a.) To help the game go quicker, Cindy and Mindy want to draw a “tree” to describe the guesses.

They start with the number 16 at the top of the tree and draw two lines (going left and right) from the number 16, and put the next “lower” and “higher” guesses:



They repeat this again. Can you draw the next line of numbers?



What will be the next line after that (draw it above)? What do you notice about all of these numbers?

Which number is missing?

b.) One characteristic of a “tree” is that there is only one path to get between any two points.

In the tree that Cindy and Mindy made, what is the “path” of numbers one must guess to get to the number 31?

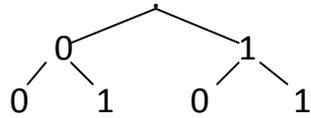
What is the path of numbers to get to the number 10?

What is the path of numbers to get to the number 21?

Can you explain why there is only one path to get to any number?



6. From a node, we add two branches, 0 to the left and 1 to the right. We do this again, like the picture below:



For each of the numbers (0 or 1) on the bottom row, write down the sequence of 0's and 1's to get to that number.

Consider these sequences as binary numbers. What are they equal to?

Draw another row of 0's and 1's and write the binary numbers that result. Can you explain why this type of tree is called a "binary tree"?

## 7. Dividing piles game

Jack and Jan each have a pile of coins. Jack has 7 coins and Jan has 8 coins. They decide to play a game with the coins. On their turn, they must divide their pile of coins, any way possible.

For example, on Jack's first turn, he can divide his pile into a pile of 1 and 6 coins, or 2 and 5 coins, or 3 and 4 coins.

On each turn, they must divide each of their piles again, any way they'd like.

The person who can no longer divide any of the piles loses. Who will always lose this game, if they play as best as they can?

## 8. Dividing chocolate game

Tony and Tina play a game. They have a piece of chocolate with 50 squares in it: 5 rows and 10 columns. Tony goes first and breaks off some number of rows and Tina goes second and breaks off some number of columns. They take turns doing this. The winner is the last person able to break off some chocolate.

Who can always win this game?