Russian Peasant Multiplication

Here is how Russian Peasant multiplication works in the case of 23×18:

```
23    18
--    --
23    18
11    36
 5    72
 2   144
 1   288
```

The following is what we need to do to complete the multiplication:

1. In the first column, take the first number and continue dividing it by 2 until you get 1.

   - If there is a remainder, drop it. For example, $23 \div 2 = 11 \ R1$, so we just write 11 in the next row.) The graphic below shows how to do this.

$$23 \ = \ 11 \times 2 \ + \ 1$$
$$11 \ = \ 5 \times 2 \ + \ 1$$
$$5 \ = \ 2 \times 2 \ + \ 1$$
$$2 \ = \ 1 \times 2 \ + \ 0$$
$$1 \ = \ 0 \times 2 \ + \ 1$$

2. In the second column, we keep doubling the second number (in this case, 18).

3. After that, we choose the rows with odd numbers in the first column, and add up the corresponding numbers in the second column.

```
            1   8
            3   6
            7   2
+     2     8   8
--   --    --  --
      4     1   4
```

Thus, the result of the multiplication is 414.

1. Multiply 13×22 using Russian Peasant Multiplication.

2. Using what you notice on the previous page, 1 do the following:

    (a) Rewrite each term in the sum: $18 + 36 + 72 + 144$ as a product of 18 and another number:

    - $18 =$

    - $36 =$

    - $72 =$

    - $288 =$

    (b) Rewrite the whole sum using the expressions above:

    - $18 + 36 + 72 + 288 =$

    (c) What do you notice? Can you simplify this expression by factoring out 18?

    Note that in this algorithm we are starting with the lowest power of 2 present in the number 23.

3. Let's learn to write numbers as sums of powers of 2 (without repetition) *starting with the smallest power of* 2 by using Russian Multiplication. Recall how Russian Peasant Multiplication works for 23×18.

Is this row used?	23	18	Multiplication Factor
			1
			2
			4
			8
			16

(a) Write the values in the column for 23 by using the following results of division by 2:

$$23 \div 2 = 11 \, R \, 1$$

$$11 \div 2 = 5 \, R \, 1$$

$$5 \div 2 = 2 \, R \, 1$$

$$2 \div 2 = 1 \, R \, 0$$

$$1 \div 2 = 0 \, R \, 1$$

(b) Write the values in the column for 18 (each next value is the double of the previous value);

(c) In the column labeled "Is this row used?" write 1 if the row is used and 0 if it is not used in Russian Peasant Multiplication. (Remember that a row is used in the Russian Peasant Multiplication if the number in the first column is odd)

(d) What do the numbers in the column "Multiplication Factor" add up to (only using rows that are used)?

(e) Write down 23 as the sum of powers of 2 starting with the smallest power:

(f) Using your answers, write down the binary representation of 23.

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4. We now have an algorithm of writing a number as a sum of powers of 2 starting with the smallest power.

–	27	–

(a) At each step, divide the number in the middle column by 2 and write the result just below it (disregarding the remainder).

(b) If the number you got in the previous step is even, write 0 in the left column. If the number is odd, write 1 in the left column.

(c) In the right column, write the powers of 2, starting with $2^0 = 1$.

(d) Circle the powers of 2 that are in the rows where the left column has 1.

(e) The circled numbers in the right column add up to the number you started with.

(f) The numbers in the left column (starting with the top number) form the binary representation of the number you started with.

5. To understand why this algorithm works, answer the following questions:

(a) What is the rightmost digit (0 or 1) in the binary representation of an even number?

(b) What is the rightmost digit (0 or 1) in the binary representation of an odd number?

6. Suppose you start with a binary number written with n digits. Using arithmetic operations, how do you get the number written by the first $(n-1)$ digits? (*Hint:* you might want to consider the cases of even and odd numbers separately). Explain the connection between the ideas you discovered in problems 5 and 6 and the algorithm of converting a number into binary notation using division by 2 that you saw in problem 4.

7. Write the following numbers as sums of powers of 2 (binary notation) starting with the smallest power (Hint: Use the method of problem 4):

(a) $9 =$

$9 \div 2 = \quad R$

(b) $14 =$

$14 \div 2 = \quad R$

(c) $23 =$

$23 \div 2 = \quad R$

(d) $44 =$

$44 \div 2 = \quad R$

8. Use the Russian Peasant multiplication in the following examples:

(a) $27 \times 23 =$

Is this row used?	27	23	Multiplication Factor

(b) $33 \times 22 =$

	33	22	

(c) $19 \times 45 =$

	19	45	

RACE!

9. Represent each number below as a sum of powers of 2. One person uses the Egyptian method (which starts with the largest power of 2 first and multiplication by 2). The other person uses the Russian Peasant method (which starts with the smallest power of 2 first and division by 2).

   (a) 26

   (b) 45

   (c) 52

   (d) 68

   (e) Which method do you like better, and why?

10. Now we know three methods of multiplication: Egyptian Multiplication, Russian Peasant Multiplication, and the "normal" long multiplication.

    (a) Give an example where Egyptian Multiplication may be easier than the other two methods. Explain why.

    (b) Give an example where Russian Peasant Multiplication may be the easiest method, and explain why.

    (c) Give an example where the regular long multiplication may be the easiest method, and explain why.