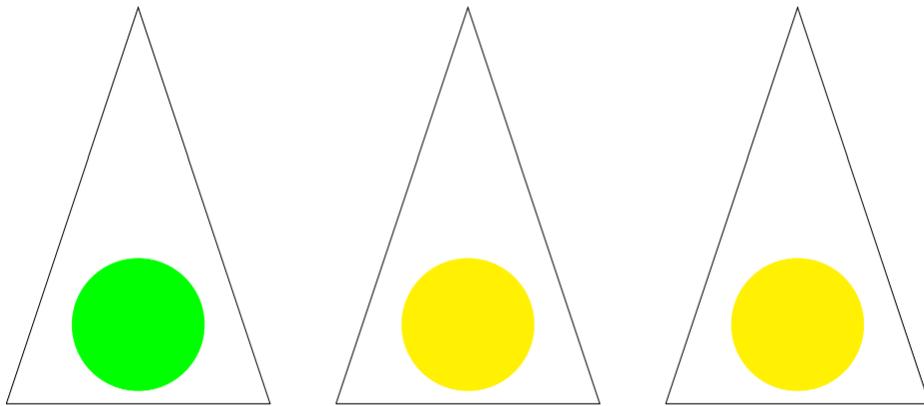


## Hats and Doors

November 20, 2016

A part of wizardry training is developing logic skills. In the following exercises, two students will be presented with three wizard's hats, one with a green label and two with yellow labels.



The students will be asked to close their eyes. One hat will be hidden, the others will be put on the students' heads. The students will open their eyes, and then be asked, "Who knows what color hat he/she has on? If so, which color is it?" The students can figure out the color hat he/she is wearing only by observing the hat of the other student. A student is not allowed to look at his/her own hat. This kind of cheating will be punished by turning the cheat into a toad!

**Problem 1** *Kaley and Emmanuelle are given two hats. Kaley gets the green-labeled hat, and Emmanuelle gets a yellow-labeled one.*

*a) Cory asks them what color hat they have on. What are Kaley and Emmanuelle going to say? Explain.*

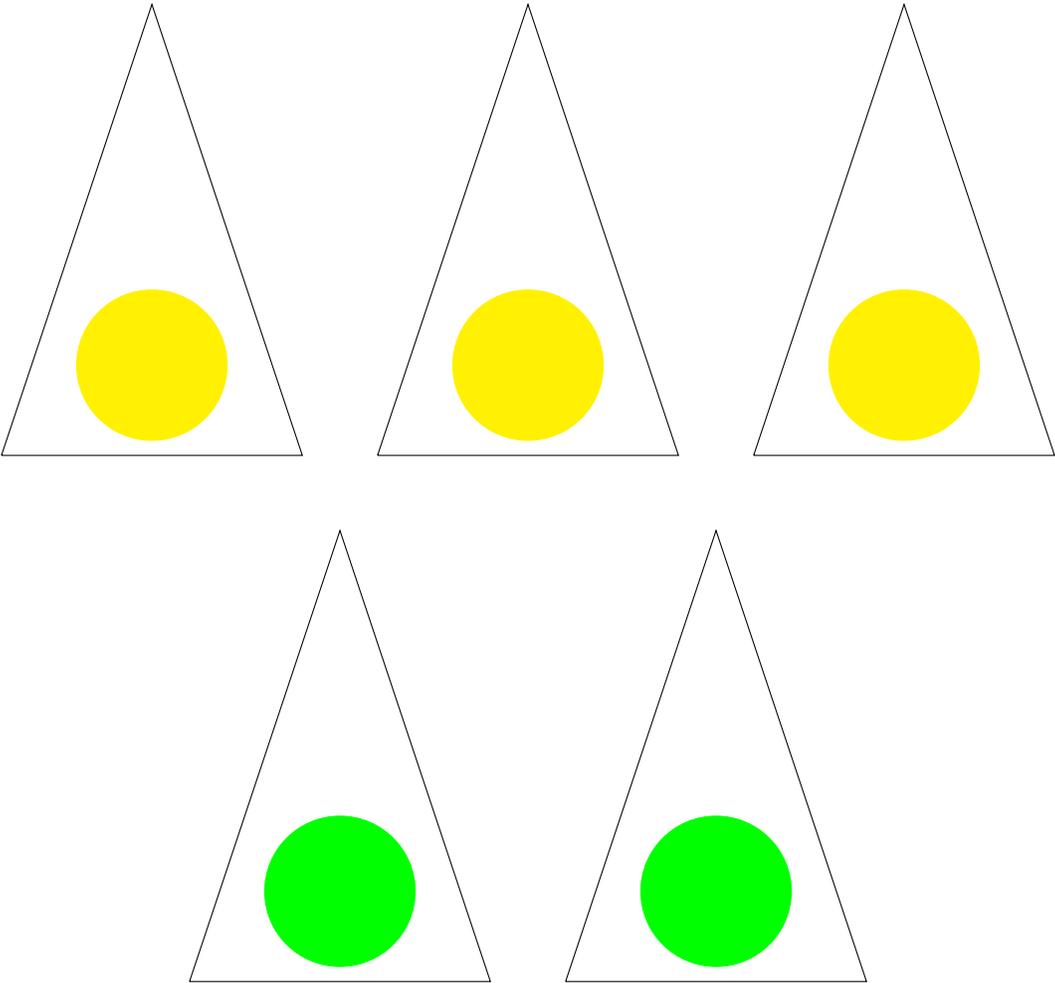
*b) After hearing Emmanuelle's answer, can Kaley now figure out which color hat she has on? Explain why or why not.*

**Problem 2** *This time, Kaley and Emmanuelle are both given the yellow-labeled hats.*

*a) Cory asks them what color hat they have on. What are Kaley and Emmanuelle going to say? Explain.*

*b) Now that they heard each other's answers, can either of them figure out the color of their hat? How?*

As the training progresses, three students will be chosen and presented with five hats, three with yellow dots and two with green. Once again, the students will be asked to close their eyes. Two hats will be hidden, three will be put on the students' heads. The students will be asked to open their eyes and to figure out the color of their hat labels by observing those of the other students.



**Problem 3** *Kaley, Emmanuelle, and Lucy are given the hats. Kaley gets a yellow-labeled hat, Emmanuelle and Lucy get the hats with green labels.*

*a) What will each person say when Cory asks them for the color of their hats? Explain.*

*b) After hearing each person's answer, can others figure out which color hat they have on? How?*

**Problem 4** *This time, Kaley and Emmanuelle get yellow-labeled hats while Lucy gets the hat with a green label.*

*a) How will each person respond to Cory's question? Explain.*

b) Now that they have heard each other's answers, who can say the color of their hat and who can't? Explain why or why not. (Hint: What would happen if one assumes that she is wearing the green hat?)

c) After hearing everybody's answers, who can now figure out the color of their hat? Explain. (Hint: What would happen if Lucy assumes she is wearing the yellow hat?)

**Problem 5** Finally, all the three students are given the yellow-labeled hats.

a) How would each person respond to Cory's question? Explain.

b) After hearing everybody's answer, who can say the color of their hat and who can't? Explain why or why not. (Hint: What would happen if one of them assumes she is wearing a green hat?)

c) After hearing each person's answer, Lucy tries to figure out the color of her hat. She initially makes the assumption that she is wearing a green hat.

i) If Lucy was wearing the green hat, would anybody have immediately known which color hat they had on? Does this agree with what you wrote for part a?

ii) After hearing everybody's answer, would anybody be able to figure out the color of their hat? (Remember, Lucy is assuming that she has a green hat on.) Does this agree with what you wrote for part b? If the answer is not the same as part b, explain what this contradiction means.

iii) Does Lucy know which color hat she has on? Does everybody else know?

There once was a king who designed the following way of punishing criminals. Convicted lawbreakers were given a choice between two doors. Behind each door, there could be either a hungry tiger or a treasure of gold, but not nothing or both. Sometimes, the king would be merciful and put gold behind both doors. However, he can be cruel and put tigers behind both doors. The king would also post some warnings on the doors and then let the criminals choose.

**Problem 6** *The king took the prisoner to the doors. There was a sign on each door. The first read, "There is gold in this room and there is a tiger in the other." The sign on the second door read, "There is gold in one of these rooms and in one of these rooms there is a tiger." "Are the signs true?" asked the prisoner. "One of them is," replied the king, "but the other is not. Now, make your choice, buddy!" Which door should the prisoner open? Why?*

**Problem 7** *For the second prisoner, the following signs were put on the doors. Door 1: at least one of these rooms contains gold. Door 2: a tiger is in the other room. "Are the signs true?" asked the prisoner. "They are either both true or both false," replied the king. Which door should the prisoner choose? Why?*

**Problem 8** *In this case, the king explained that, again, the signs were either both true or both false. Sign 1: either this room contains a tiger, or there is gold in the other room. Sign 2: there is gold in the other room. Does the first room contain gold or a tiger? What about the other room?*

Once upon a time, in a land far, far away there lived a very beautiful princess, the only daughter of a very evil king. Some day, a handsome and very smart prince from a neighboring kingdom came to pay them a visit.

The princess and prince fell in love with each other and asked the king for a permission to marry. The evil king didn't want his daughter to leave. Instead of blessing the marriage, he ordered to put the prince in jail and to prepare for his exile.

The princess begged the king not to kill the prince and finally he agreed. He told the prisoner, "At her Highness's request, I will give you a chance. Tomorrow you will be brought to my court. You will have to pull a lot. I will put two pieces of paper in the box. One will read MARRIAGE, the other will read EXILE. Whatever piece you pull out, it will be your destiny."

The king was a very evil man. He ordered his minister to write EXILE on both pieces of paper. The princess overheard the king's order and found a way to warn the prince.

**Problem 9** *What should the prince do to survive?*

*Hint: kings do not like public embarrassment.*