November 6, 2016

Fake coins, map coloring, and clock arithmetic!

In the problems below, each real coin weighs 10 grams. Each fake coin weighs 9 grams. The scale we use is a digital scale, not a two-pan balance.

1. Suppose you have two bags full of coins. One of the bags has entirely fake coins, and the other bag has entirely real coins.

   How can you determine which bag has real coins and which has fake coins *in just one weighing*?

   (a) Suppose you put the coin(s) from only one of the bags on the scale. Will you be able to determine which bag contains fake coins?

(b) Suppose you put one coin from each of the bags on the scale. Will you be able to determine which bag contains fake coins?

(c) Decide how many coins from each of the bags you need to put on the scale. Describe the possible outcomes (how much could the coins weigh together), and explain how you can determine which bag has the fake coins.

2. Now suppose you have 3 bags full of coins. Again, one of the bags holds fake coins. How can you determine which bag has fake coins in just one weighing? Describe what you will put on the scale and how you will decide where the fake coins are.

3. Suppose now you have 10 bags of coins. One of the bags is filled with fake coins. Can you figure out which bag has the fake coins in just one weighing?

Find the bag with the fake coins

We have just attempted to solve the following problem:

Given 10 bags full of coins (one bag containing fake coins, and the other bags containing real coins), find out which bag contains the fake coins in just one weighing using a digital scale. (Note that a fake coin weights 9 grams while a real one weights 10 grams).

1. One way to solve the problem is to put the following combination of coins on the digital scale:

   - 1 coin from the first bag
   - 2 coins from the second bag
   - 3 coins from the third bag
   - . . .
   - 10 coins from the tenth bag

   (a) How many coins in all are you placing on the scale?

   (b) If all the coins were real, how much would the coins weigh all together?

Given that one of the bags contains fake coins, make a table showing what the total possible weight of the coins is and how this tells you which bag contains the fake coins.

Number of Fake Coins from a bag	Total Weight
1	549

2. Rita, a Kindergarten student, placed some number coins from all of the bags on the scale:

- Exactly five of the coins are from bag 3
- Exactly five of the coins are from bag 4
- The rest of the coins are from other bags

(a) She thinks she can always determine which of the bags contains the fake coins in just one weighing. Is Rita right?

(b) Can she get lucky and be able to determine what bag contains the fake coins? If yes, explain how this can happen.

(c) What happens when the fake coins are in bag 3 or 4?

3. Tyler, a Junior circle student, thinks that it is enough to place the coins from only 9 bags on the digital scale in order to find out which of the bags contains the fake coins. Do you think he is right? If yes, explain what he has in mind. If not, explain why not.

Calendar problems

1. Today is a thursday. Kate's birthday is in 5 days. What day of the week is Kate's birthday?

2. Three days ago, yesterday was the day before Sunday. What day will it be tomorrow?

3. Thomas has to write a long essay for his English class. He got this assignment on Tuesday, March 12th. The assignment is due on Thursday, April 18th. Thomas plans to work on the essay every Monday and every Friday. How many days does he allow himself to work on the essay? (Note there are 31 days in March.)

Map coloring problems

Color the following "map" or pattern *using as few colors as possible*.
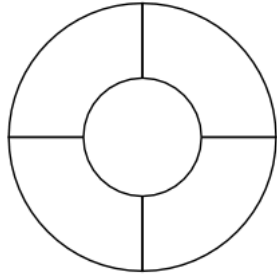
The only rule is this:

Two regions that share a common edge cannot be colored the same!
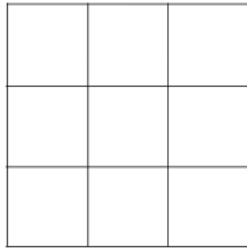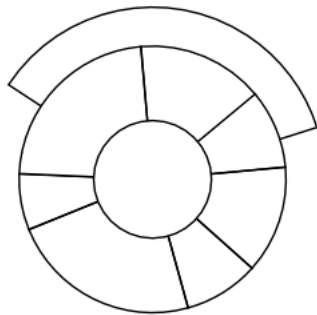

How many colors did it take?

(To save time, instead of actually coloring, just label each region with the name of a color such as "red" or "green" or "blue".)

Now color the following map using as few colors as possible.

How many colors did it take this time?

Here is a more challenging map:

How many colors does it take to color this one?

How about this one?

Can you draw a "map" or pattern that requires more colors than the examples given above?
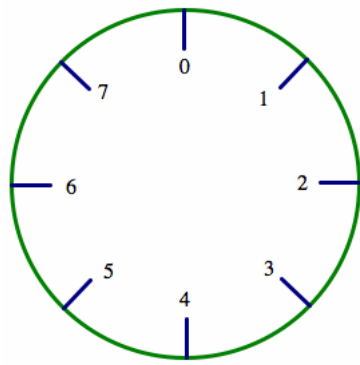
Remember:

Regions that share a boundary cannot be colored the same color!

Clock arithmetic

1. A clock says that it's 7 pm. What time will the clock say in 30 hours?

2. A clock says it's 5 pm. What time will the clock say in 130 hours?

3. A clock says it's 7 pm. What time did the clock say 100 hours ago?

4. Imagine there were only 8 hours in a day. Clocks would look like this:

If the hour hand on this clock is pointing at 5, where will it be pointing in 70 hours?

5. If the hour hand on the clock in the previous question is pointing at 2, where was it pointing 1000 hours ago?