Egyptian Multiplication

Junior Circle 10/23/2016

Warmup

What does the exponent operator do? Can you write 3^5 in terms of another operation you already know? How about a^b.

List the first 10 powers of 2 (i.e. 2^0, 2^1, 2^2, etc). Put them in exponent form as well as in normal, integer form.

Ancient Egyptians had an interesting method for multiplying two numbers. Suppose that you have to multiply two numbers (e.g., 23 and 18). The basic operation for them was multiplying a number by 2. (In other words, adding a number to itself). They reduced all other multiplication problems to it. Here is how they would start multiplying 23 by 18 (in modern notation):

$$
\begin{array}{cc}
23 & 18 \\
-- & -- \\
\mathbf{1} & \mathbf{18} \\
\mathbf{2} & \mathbf{36} \\
\mathbf{4} & \mathbf{72} \\
8 & 144 \\
\mathbf{16} & \mathbf{288}
\end{array}
$$

Here is what they did to complete the multiplication

1. Below the first number (in this case, 23), they would write all of the powers of 2 that are smaller or equal to the number.

2. In the second column, they would keep doubling the second number (in this case, 18).

3. After that, they would represent the first number as the sum of the powers of 2 (so that each of the powers of 2 is used at most once).
   For example, if the first number is 23, they would find

$$23 = 16 + 4 + 2 + 1.$$

   After that, they would mark those rows where these powers of 2 are present in the left column. (In our example, the first, the second, the third, and the fifth rows are marked).

Finally, all there is to do at this point is to add the marked numbers in the second column:

$$
\begin{array}{ccc}
 & 1 & 8 \\
 & 3 & 6 \\
 & 7 & 2 \\
+ \quad 2 & 8 & 8 \\
-- & -- & -- \\
4 & 1 & 4
\end{array}
$$

Thus, the result of the multiplication is 414.

The goal of our session today is to understand how and why the Egyptian's method of multiplication worked.

1. Would the following sum of powers of 2 be used to represet the value 38 when using Egyptian multiplication?

$$38 = 2 + 4 + 16 + 16$$

    (a) Yes or No. (Circle one.)

    (b) Why?

    (c) Correct the problem.

2. Represent each of the numbers below as a sum of powers of 2, using each power of 2 at most once.

    (a) $9 =$

    (b) $14 =$

    (c) $23 =$

    (d) $44 =$

3. Multiply the following numbers using Egyptian Multiplication:

(a) 13×41

Complete the chart:

13	41

Compute the addition:

(b) 41×13

Complete the chart:

41	13

Compute the addition:

4. Because multiplication is commutative, there are two ways to apply the Egyptian Multiplication algorithm: Suppose $a < b$. Multiply $a \times b$ or $b \times a$. Which would you prefer? Why? Consider the example 17×35 vs. 35×17.

5. Give your own explanation of how Egyptian Multiplication works.

6. In your opinion, is it easier to use the regular multiplication strategy or Egyptian Multiplication?

7. With a partner, have a race to see who can multiply numbers faster. One of you must use Egyptian Multiplication and the other must use regular, long multiplication. Race 4 times switching the type of multiplication you do. Show your work below:

   (a) $25 \times 31 =$

   (b) $38 \times 45 =$
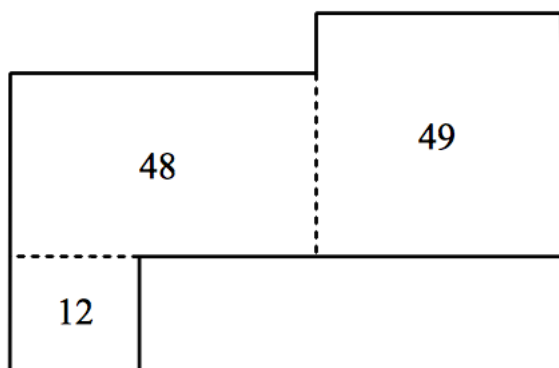
   (c) $12 \times 63 =$

   (d) $17 \times 52 =$

8. Below are shapes that are divided into different rectangles. The area for each rectangle is given. Your job is to label the lengths of all the sides as well as to give the total perimeter.

(a)

10

14

Perimeter =

(b)

49

48

12

Perimeter =

9. Writing numbers in binary (how computers read and communicate) is intimately related to powers of 2. Normally, we write our numbers in a base 10 system. You can see this as every time we reach a new power of 10, we add a new digit to our numbers. For example, $10^0 = 1$ is 1 digit, $10^1 = 10$ has 2 digits, $10^2 = 100$ has 3 digits, and so on. In binary, instead of powers of 10 adding new digits, powers of 2 now add new digits. Similarly, $2^0 = 1_2$ has 1 digit, $2^1 = 2 = 10_2$ has 2 digits, $2^2 = 4 = 100_2$ has 3 digits, even though 10_2 and 100_2 actually represent our normal numbers of 2 and 4. Some examples of number written in binary are $3 = 11_2$, $6 = 110_2$, $11 = 1011_2$, and $8 = 1000_2$. Now write the below numbers in binary.

   (a) 7

   (b) 9

   (c) 15

   (d) 18

   (e) 25

(f) 32

(g) 22

(h) 37

(i) 44

(j) 66

(k) 97