

Math Circle
Intermediate Group
October 30, 2016
Geometric Probability

Warm up problems

1. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using Venn diagrams.

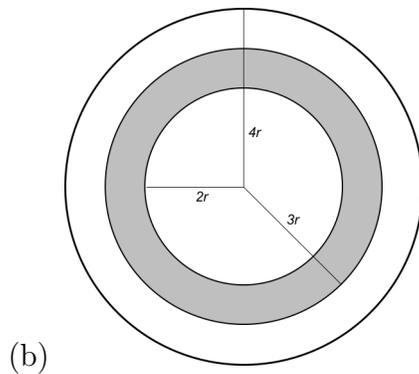
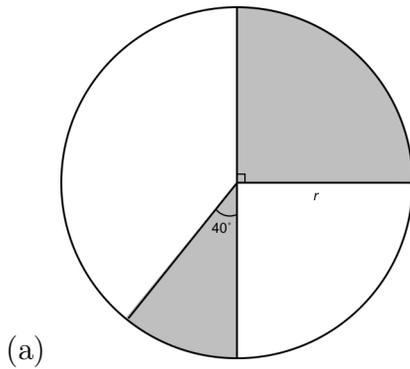
2. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using Venn diagrams.

Geometric Probability¹

The probability of an event is the likelihood of the event to occur. With geometric probability, we are looking for the likelihood that we will hit a particular area of a figure. It can be calculated as the ratio of the desired area to total area.

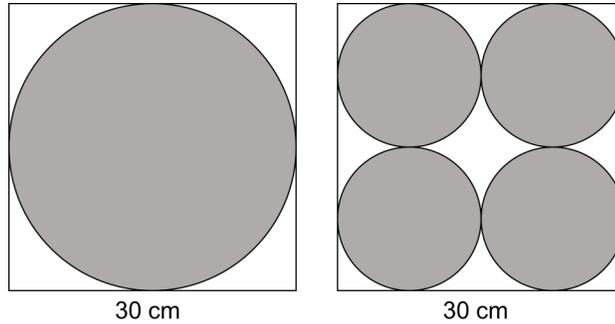
$$\text{Probability of hitting desired area} = \frac{\text{Size of desired region}}{\text{Size of total region}}$$

1. In each figure below, find the probability that a randomly chosen point lies in the shaded region.



¹Some problems in this section have been taken from the “Art of Problem Solving” by J. Batterson.

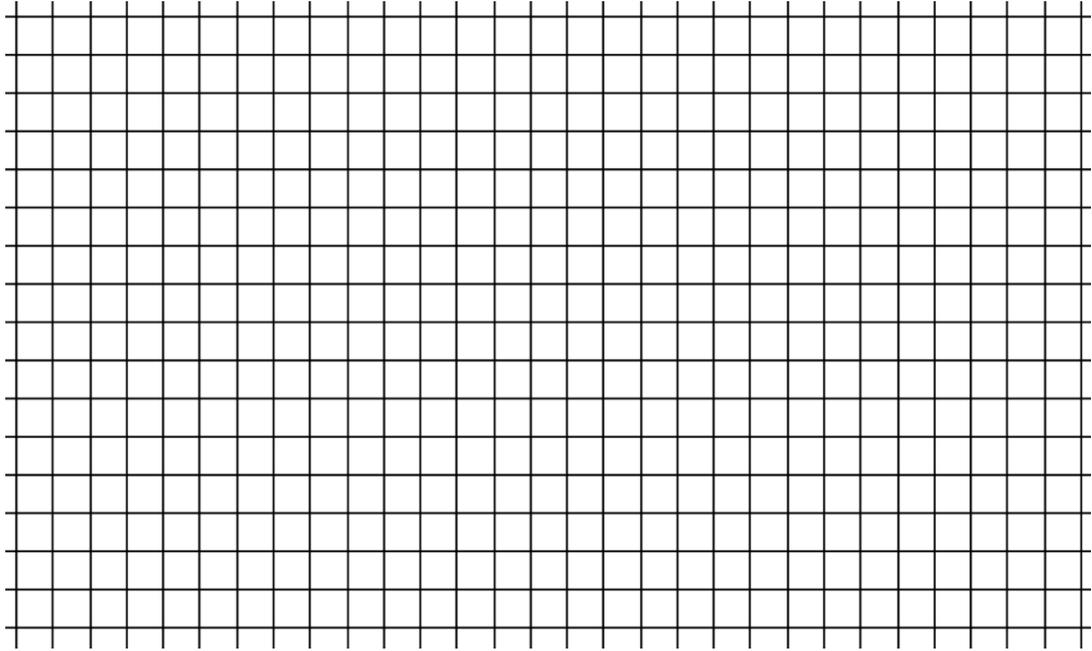
2. BonSoon and Jordan are both designing a dartboard. The diagrams below show the pattern each student prefers for his dartboard design. Which pattern has a greater probability for a contestant to throw a dart into the shaded area?



3. Ivy is driving a car in a line of cars, with about 100 feet between successive cars. Each car is 15 feet long. At the next overpass, there is a large icicle. The icicle is about to crash down onto the highway. If the icicle lands on or within 20 feet of the front of a car, it will cause an accident. What is the chance that the icicle will cause an accident?

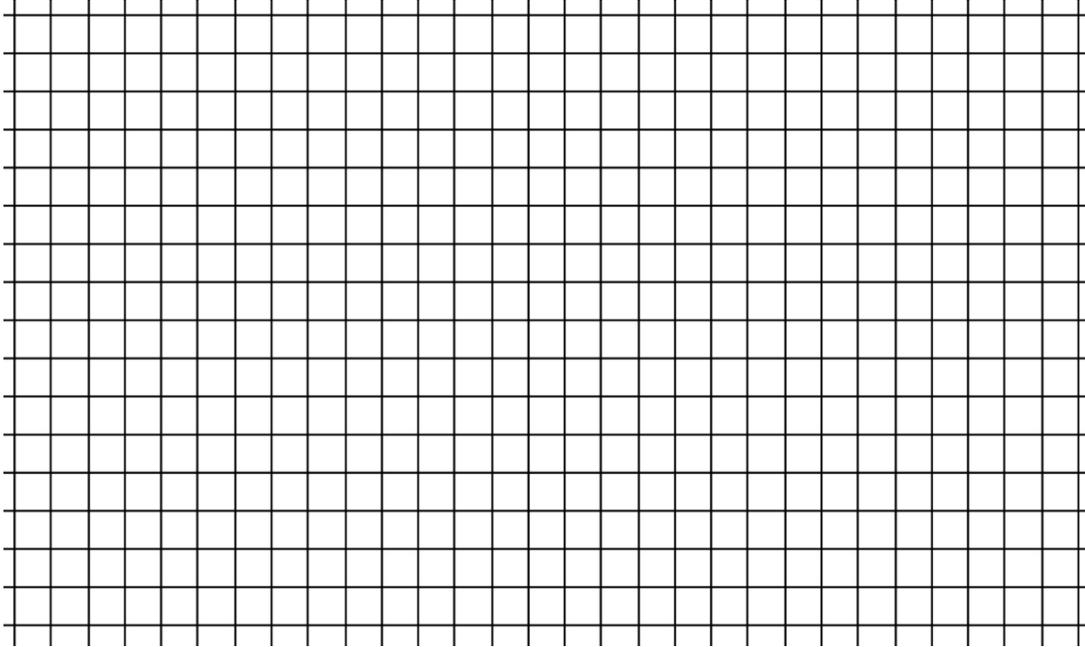
4. The bus comes to a stop near your house every morning at a random time between 6:45 and 6:50. You arrive at the bus stop at a random time between 6:40 and 6:45 every day and wait until the bus comes. What is the probability that you wait less than 4 minutes for the bus to arrive?

(a) Set up a graph with your arrival time graphed against the arrival time of the bus. Shade the area where your wait time is less than 4 minutes.



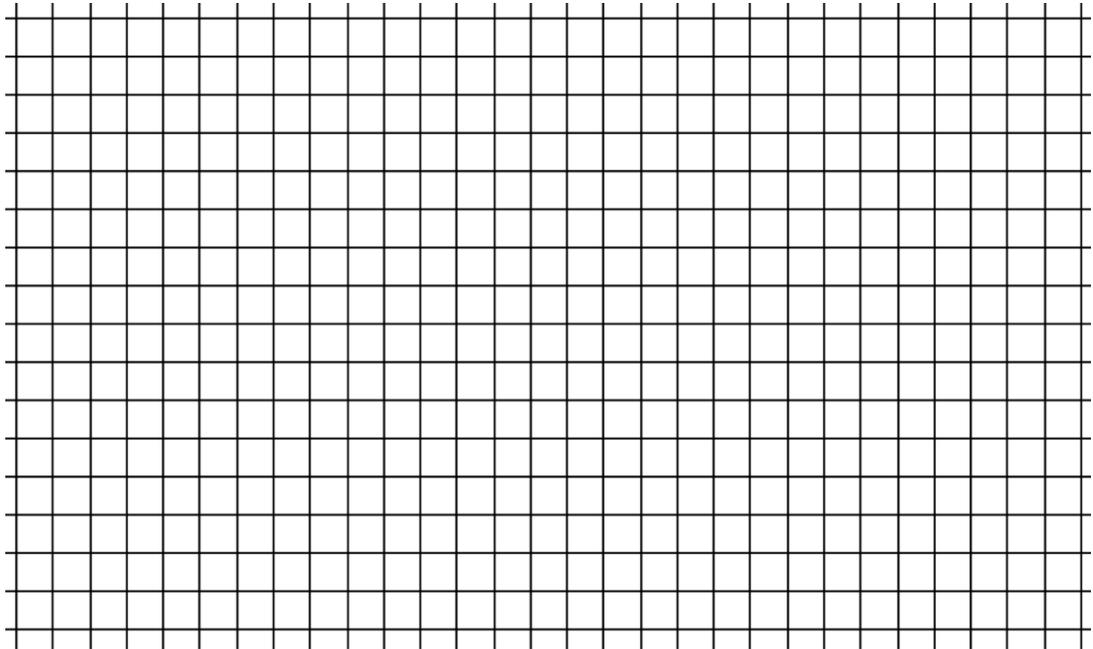
(b) Find out the area of the shaded region and calculate the probability.

5. Two friends, who take the train to their jobs from the same station, arrive at the station randomly between 8 and 8:20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train together or by themselves. What is the probability that they will take the train together?



6. What is the probability that two whole numbers selected at random between 1 and 4 (including 1 and 4) have a sum greater than 4? Express your answer as a common fraction in the simplest form.

(a) What if the two numbers selected at random do not have to be whole numbers?



7. A rectangular prism has edges of length 3, 4 and 5 cm. What is the probability that a randomly selected point on the surface of the prism will be on one of its two smallest faces?

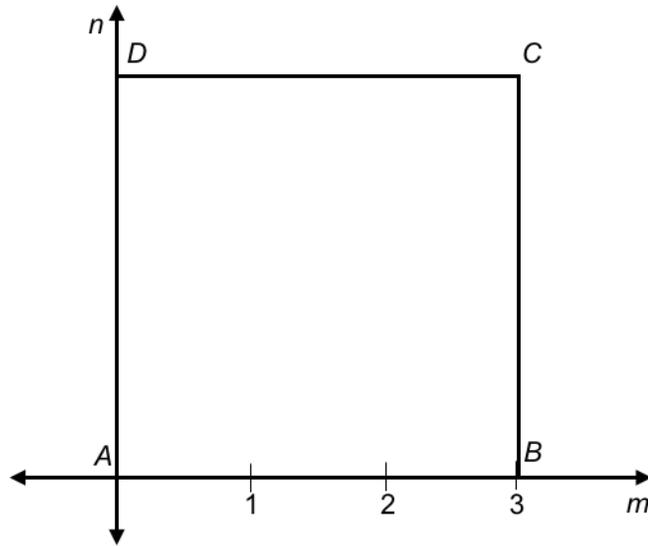
8. Point X is selected at random within square $ABCD$ of side length 3. What is the probability that quadrilateral $ABXD$ has an area greater than 4 square units?

(a) Draw a square $ABCD$ below, and mark any point X in it. Join the points A , B and D to X to get triangles ADX and ABX .

(b) We can consider quadrilateral $ABXD$ as the sum of the areas of triangles ADX and ABX . What is the base length of each triangle?

- (c) Assume that the height of triangle ADX is m and the height of triangle ABX is n . If the area of quadrilateral $ABXD$ must be greater than 4 square units, what do we know about the relationship between m and n ?

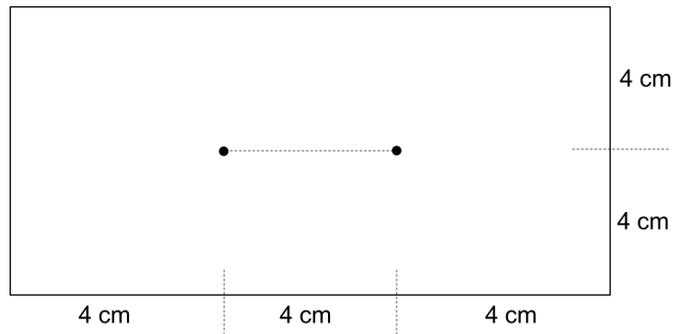
- (d) Graph that relationship below and shade the area within which X must lie.



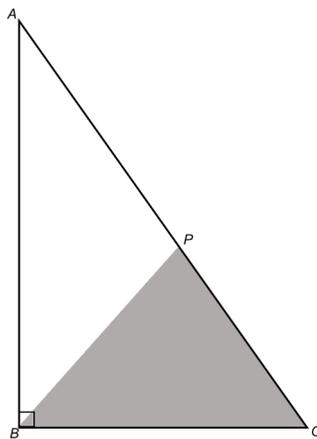
- (e) Find the probability that X lies in the shaded region.

9. In rectangle $ABCD$, $AB = 1$ and $BC = 2$. Point X is selected at random within the rectangle. What is the probability that the area of triangle ABX is more than twice the area of triangle BCX ?

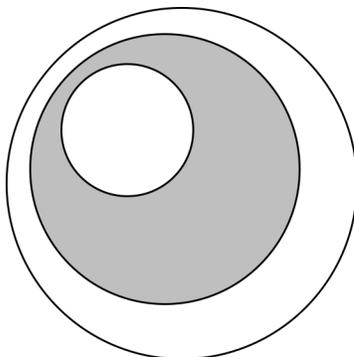
10. Two points are indicated below within a rectangle of side lengths 8 and 12 units. Point X is randomly selected within the rectangle. What is the probability that the triangle formed will be an obtuse triangle?



11. A point is randomly selected inside the right triangle ABC . What is the probability that it will fall within the shaded region if $AP = 3$ and $CP = 2$?



12. Circles of integral diameter are arranged so that each is entirely within the next larger circle, and the probability of a randomly selected point being within the shaded region is exactly $1/2$. What is the smallest possible area of the shaded region?



13. One side of a triangle is 5 cm long. Two (not necessarily integer) numbers are randomly selected between 0 and 10. What is the probability that the two numbers can be the other two sides of the triangle? (Hint: In a triangle, the sum of the lengths of any two sides must be greater than the length of the third side.)

